TOBB ETÜ ELE 273 ELEKTRİK MÜHENDİSLERİ İÇİN OLASILIK KURAMI

FORMÜLLER

Bernoulli (p): $0 \le p \le 1$

$$P_X(x) = \begin{cases} 1-p & x=0 \\ p & x=1 \\ 0 & diğer \end{cases}$$

$E[X]=p \qquad Var[X]=p(1-p)$

Poisson(α): α>0

$$P_X(x) = \begin{cases} \frac{\propto^x e^{-\propto}}{x!} & x=0,1,2,\dots \\ 0 & diğer \end{cases}$$

$E[X]=\propto \qquad Var[X]=\propto$

Binomial (n,p): $0 \le p \le 1$ ve n pozitif tamsayı

$$P_X(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$E[X]=np \qquad Var[X]=np(1-p)$

Üstel- Exponential(λ): λ>0

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \ge 0 \\ 0 & diğer \end{cases}$$

$E[X]=\frac{1}{\lambda} \qquad Var[X]=\frac{1}{\lambda^2}$

Discrete Uniform(k,l): k,l tamsayı ve k<l

$$P_X(x) = \begin{cases} \frac{1}{l-k+1} & x=k,k+1,\dots,l \\ 0 & diğer \end{cases}$$

$E[X]=\frac{k+l}{2} \qquad Var[X]=\frac{(l-k)(\iota-l+2)}{12}$

Düzgün- Uniform(a,b): a<b

$$f_X(x) = \begin{cases} \frac{1}{b-a} & a<x<b \\ 0 & diğer \end{cases}$$

$E[X]=\frac{a+b}{2} \qquad Var[X]=\frac{(b-a)^2}{12}$

Geometric(p): $0 \le p \le 1$

$$P_X(x) = \begin{cases} p(1-p)^{x-1} & x=1,2,\dots \\ 0 & diğer \end{cases}$$

$E[X]=\frac{1}{p} \qquad Var[X]=\frac{(1-p)}{p^2}$

Erlang(n,λ): λ>0 ve n pozitif tamsayı

$$f_X(x) = \begin{cases} \frac{\lambda^n x^{n-1} e^{-\lambda x}}{(n-1)!} & x \ge 0 \\ 0 & diğer \end{cases}$$

$E[X]=\frac{n}{\lambda} \qquad Var[X]=\frac{n}{\lambda^2}$

Pascal(k,p): $0 \le p \le 1$, k pozitif tamsayı

$$P_X(x) = \binom{x-1}{k-1} p^k (1-p)^{x-k}$$

$E[X]=\frac{k}{p} \qquad Var[X]=\frac{k(1-p)}{p^2}$

Gaussian(μ,σ): σ>0 ve -∞<μ<∞

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$E[X]=\mu \qquad Var[X]=\sigma^2$

Bivariate Gaussian($\mu_X,\mu_Y,\sigma_X,\sigma_Y,\rho_{X,Y}$): $\sigma_X>0,\sigma_Y>0,-\infty<\mu_X<\infty,-\infty<\mu_Y<\infty,-1<\rho_{X,Y}<1$

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho_{X,Y}^2}} e^{-\frac{\left(\frac{x-\mu_X}{\sigma_X}\right)^2 - \frac{2\rho_{X,Y}(x-\mu_X)(y-\mu_Y)}{\sigma_X\sigma_Y} + \left(\frac{y-\mu_Y}{\sigma_Y}\right)^2}{2\left(1-\rho_{X,Y}^2\right)}}$$

$$E[Y|X=x] = \mu_Y + \rho_{X,Y}\frac{\sigma_Y}{\sigma_X}(x-\mu_X), \qquad \sigma_{Y|X=x} = \sigma_Y\sqrt{1-\rho_{X,Y}^2}$$

$cos(A+B)=\cos(A)\cos(B)-sin(A)\sin(B)$

$\sin(A+B)=\sin(A)\cos(B)+\cos(A)\sin(B)$

$sin(A)\sin(B)=\frac{1}{2}[\cos(A-B)-\cos(A+B)]$

$cos(A)\cos(B)=\frac{1}{2}[\cos(A-B)+\cos(A+B)]$

$sin(A)\cos(B)=\frac{1}{2}[\sin(A+B)+\sin(A-B)]$

$e^{j\theta}=\cos(\theta)+jsin(\theta)$

$\cos(\theta)=\frac{1}{2}\left[e^{j\theta}+e^{-j\theta}\right], \sin(\theta)=\frac{1}{2j}\left[e^{j\theta}-e^{-j\theta}\right]$

$\int_a^b udv = uv\Big|_a^b - \int_a^b vdu$

$\sum_{i=0}^n q^i = 1+q+q^2+\cdots+q^n = \frac{1-q^{n+1}}{1-q}$

$\sum_{i=1}^\infty iq^i = \frac{q}{(1-q)^2}, \quad \sum_{i=0}^\infty q^i = \frac{1}{1-q}$ (eğer |q|<1)

$\sum_{i=1}^n iq^i = \frac{q(1-q^n[1+n(1-q)])}{(1-q)^2}$

$\sum_{i=1}^n i = \frac{n(n+1)}{2}$

$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$

$E[g(X)] = \int_{x\in S_X} g(x)f_X(x)dx$

$E[g(X)] = \sum_{x\in S_X} g(x)P_X(x)$

$Var[X]=E[X^2]-E[X]^2$

$Q(x)=\int_x^\infty \frac{1}{\sqrt{2}} e^{-\frac{x^2}{2}}dx \qquad \Phi(x)=1-Q(x)$

Bağımsız $\leftrightarrow f_{XY}(x,y)=f_x(x)f_Y(y)$

$P_{XY}(x,y)=P_x(x)P_Y(y)$

$f_X(x)=\int_{y\in S_Y} f_{XY}(x,y)dy$

$P_X(x)=\sum_{y\in S_Y} P_{XY}(x,y)$

$F_{X|B}(x)=P[X\le x|B], x\in B$

$f_X(x)=\sum_{i=1}^m f_{X|B_i}(x)P[B_i], B_1,\dots B_m\ partition$

$E[E[X|Y]]=E[X]$

$E[X|B]=\int_{-\infty}^{+\infty} xf_{X|B}(x)dx$

$E[X]=\sum_{i=1}^m E[X|B_i]P[B_i]$

$P_{X|Y}(x|y)=\frac{P_{X,Y}(x,y)}{P_Y(y)}$

$f_{Y|X}(y|X=x)=\frac{f_{XY}(x,y)}{f_X(x)}, x\in S_X$

Sample mean: $M_n(X)=\frac{X_1+X_2+\cdots+X_n}{n}$

Chebyshev: $P\left[\left|Y-\mu_y\right|\ge c\right]\le\frac{Var[Y]}{c^2}$