MIMO-OFDM Wireless Communications with MATLAB®

min (N_R, N_T)

Spatial Multiplexing

Chapter 11. Uzamsal Çoklamalı MIMO Sistemleri

Yong Soo Cho | Jaekwon Kim
Won Young Yang | Chung G. Kang

WILEY

Chapter 11. Signal Detection for Spatially Multiplexed MIMO Systems

Bu konudaki varsayım

$\boxed{\text{CSIR}}$

Sadece alıcıda kanal bilgisi olduğu varsayımı

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  - 11.1.1 ZF Signal Detection
  - 11.1.2 MMSE Signal Detection
- 11.2 OSIC SIGNAL DETECTION (Linear)
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$\boxed{\text{Verici her anteninden farklı bilgi basar.}}$

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Chapter 11. Uzamsal Çoklamalı MIMO Sistemleri

$N_R \times N_T$ $N_T \times 1$ vector $\to [x_1, x_2, ---, x_{N_T}]^T$

$N_R \times 1$ noise $[z_1, z_2, --- z_{N_R}]^T \sim \mathcal{N}(0, \sigma_z^2)$

rx $[y_1, y_2, .. y_{N_R}]^T$

$$\begin{aligned} \mathbf{y} &= \mathbf{H}\mathbf{x} + \mathbf{z} \\ &= \mathbf{h}_1 x_1 + \mathbf{h}_2 x_2 + \cdots + \mathbf{h}_{N_T} x_{N_T} + \mathbf{z} \end{aligned} \tag{11.1}$$

$N_R \times 1$ $N_R \times 1$ $N_R \times 1$ skaler

11.1 Linear Signal Detection

istenen sinyal dışında her şey gürültü muamelesi görür

$$\tilde{\mathbf{x}} = [\tilde{x}_1 \tilde{x}_2 \cdots \tilde{x}_{N_T}]^T = \mathbf{W}\mathbf{y}, \tag{11.2}$$

$N_R \times 1$

$N_T \times N_R$ weight matrix

11.1 Linear Signal Detection

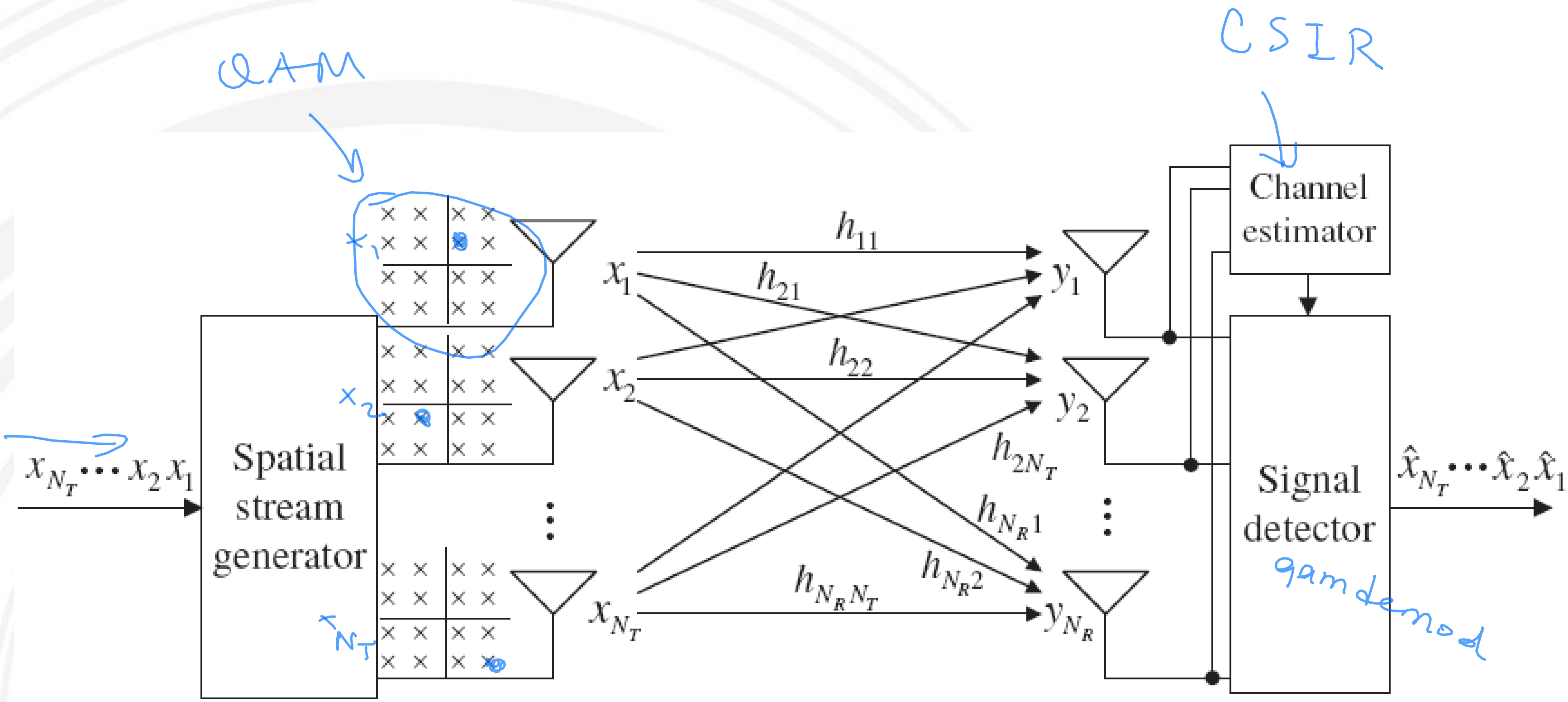

Figure 11.1 Spatially multiplexed MIMO systems.

11.1.1 ZF Signal Detection

(Zero Forcing)

pseudo inverse (neden gerçek inv kullanmıyoruz)

weight matrix

$$\mathbf{W}_{ZF} = \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H. \tag{11.3}$$

$H_{N_R \times N_T}$

$\bar{y} = \bar{H}\bar{x} + \bar{z}$

$\bar{W}\bar{y} = \bar{W}\bar{H}x + \bar{W}\bar{z}$

rx

$$\begin{aligned}
\tilde{\mathbf{x}}_{ZF} &= \mathbf{W}_{ZF}\mathbf{y} \\
&= \mathbf{x} + \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H\mathbf{z} \\
&= \mathbf{x} + \tilde{\mathbf{z}}_{ZF}
\end{aligned} \tag{11.4}$$

$= \underbrace{(\bar{H}^H\bar{H})^{-1}\bar{H}^H\bar{H}}_{I}\bar{x} + (\bar{H}^H\bar{H})^{-1}\bar{H}^H\bar{z}$

$\bar{W}\bar{y} = \bar{x} + \underbrace{(\bar{H}^H\bar{H})^{-1}\bar{H}^H\bar{z}}_{\tilde{z}}$

$N_T \times N_R$ $\quad N_R \times 1$ $\quad$ tx symbols $N_T \times 1$

gürültü gücü

$$\begin{aligned}
\left\|\tilde{\mathbf{z}}_{ZF}\right\|_2^2 &= \left\|\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H\mathbf{z}\right\|^2 \\
&= \left\|\left(\mathbf{V}\mathbf{\Sigma}^2\mathbf{V}^H\right)^{-1}\mathbf{V}\mathbf{\Sigma}\mathbf{U}^H\mathbf{z}\right\|^2 \\
&= \left\|\mathbf{V}\mathbf{\Sigma}^{-2}\mathbf{V}^H\mathbf{V}\mathbf{\Sigma}\mathbf{U}^H\mathbf{z}\right\|^2 \\
&= \left\|\mathbf{V}\mathbf{\Sigma}^{-1}\mathbf{U}^H\mathbf{z}\right\|^2
\end{aligned} \tag{11.5}$$

I

$= \|\Sigma^{-1}\bar{U}^H\bar{z}\|$

$(H = U\Sigma V^H$ S.V.D.$)$

unitary $\quad$ unitary

$\Sigma \in \mathbb{R}^{N_R}$

$\|Qx\| = \|x\|$ eğer Q unitary ise

11.1.1 ZF Signal Detection

$$
\begin{aligned}
E\left\{\|\tilde{\mathbf{z}}_{ZF}\|_2^2\right\} &= E\left\{\left\|\boldsymbol{\Sigma}^{-1}\mathbf{U}^H\mathbf{z}\right\|_2^2\right\} \\
&= E\left\{\mathrm{tr}\left(\boldsymbol{\Sigma}^{-1}\mathbf{U}^H\mathbf{z}\mathbf{z}^H\mathbf{U}\boldsymbol{\Sigma}^{-1}\right)\right\} \\
&= \mathrm{tr}\left(\boldsymbol{\Sigma}^{-1}\mathbf{U}^H E\{\mathbf{z}\mathbf{z}^H\}\mathbf{U}\boldsymbol{\Sigma}^{-1}\right) \\
&= \mathrm{tr}\left(\sigma_z^2\boldsymbol{\Sigma}^{-1}\mathbf{U}^H\mathbf{U}\boldsymbol{\Sigma}^{-1}\right) \\
&= \boxed{\sigma_z^2\,\mathrm{tr}\left(\boldsymbol{\Sigma}^{-2}\right)} \\
&= \boxed{\sum_{i=1}^{N_T}\frac{\sigma_z^2}{\sigma_i^2}}.
\end{aligned}
\qquad (11.6)
$$

$= \mathrm{tr}\left((\Sigma^{-1}U^H z)(\Sigma^{-1}U^H z)^H\right)$

$\tilde{z}$

$\frac{\sigma_z^2}{\min_i(\sigma_i^2)}$

$\sigma_z^2 I$

I

diag

$\begin{bmatrix} \sigma_1 & & & 0 \\ & \sigma_2 & & \\ & & \ddots & \\ 0 & & & \sigma_{N_T} \end{bmatrix}^{-2}$

ZF

singular value düşük ise gürültü büyüyor.

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11.1.2 MMSE Signal Detection

(ZF'den daha iyi)

Minimum Mean Square Error

weight matrix

$$\mathbf{W}_{MMSE} = \left(\mathbf{H}^H\mathbf{H} + \sigma_z^2\mathbf{I}\right)^{-1}\mathbf{H}^H. \qquad (11.7)$$

} $N_T \times N_R$

$N_T \times N_T$ $\quad$ $N_T \times N_R$

gürültünün varyansını (gücünü) bilmeyi gerektirir.

bunu maksimize eder.

$$\mathbf{w}_{i,MMSE} = \underset{\mathbf{w}=(w_1,w_2,\ldots,w_{N_T})}{\arg\max} \frac{|\mathbf{wh}_i|^2 \mathsf{E}_x}{\mathsf{E}_x \sum_{j=1,j\neq i}^{N_T} |\mathbf{wh}_j|^2 + \|\mathbf{w}\|^2\sigma_z^2} \qquad (11.8)$$

$1 \times N_R$

i'inci satırı.

$$\begin{aligned}
\tilde{\mathbf{x}}_{MMSE} &= \mathbf{W}_{MMSE}\mathbf{y} \\
&= \left(\mathbf{H}^H\mathbf{H} + \sigma_z^2\mathbf{I}\right)^{-1}\mathbf{H}^H\mathbf{y} \\
&= \tilde{\mathbf{x}} + \left(\mathbf{H}^H\mathbf{H} + \sigma_z^2\mathbf{I}\right)^{-1}\mathbf{H}^H\mathbf{z} \\
&= \tilde{\mathbf{x}} + \tilde{\mathbf{z}}_{MMSE}
\end{aligned} \qquad (11.9)$$

$\neq x$

$\bar{x}$ değil.

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11.1.2 MMSE Signal Detection

SVD

$(H = \bar{U} \Sigma \bar{V}^H)$

$$\|\tilde{\mathbf{z}}_{MMSE}\|_2^2 = \left\| \left(\mathbf{H}^H\mathbf{H} + \sigma_z^2\mathbf{I}\right)^{-1}\mathbf{H}^H\mathbf{z} \right\|^2$$
$$= \left\| \left(\mathbf{V}\mathbf{\Sigma}^2\mathbf{V}^H + \sigma_z^2\mathbf{I}\right)^{-1}\mathbf{V}\mathbf{\Sigma}\mathbf{U}^H\mathbf{z} \right\|^2. \tag{11.10}$$

$(\bar{V}\bar{\Sigma}^2\bar{V}^H + \sigma_z^2 I)^{-1} V\Sigma = (\bar{V}\bar{\Sigma}^2\bar{V}^H + \sigma_z^2 \bar{I})^{-1} (\Sigma^{-1}V^H)^{-1}$

$(V\Sigma^{-1})(\bar{V}\bar{\Sigma}^2 V^H + \sigma_z^2 I)$

$$\|\tilde{\mathbf{z}}_{MMSE}\|_2^2 = \left\| \left(\mathbf{\Sigma}\mathbf{V}^H + \sigma_z^2\mathbf{\Sigma}^{-1}\mathbf{V}^H\right)^{-1}\mathbf{U}^H\mathbf{z} \right\|^2 = \left\| \mathbf{V}\left(\mathbf{\Sigma} + \sigma_z^2\mathbf{\Sigma}^{-1}\right)^{-1}\mathbf{U}^H\mathbf{z} \right\|^2 \tag{11.11}$$

→ bunu atabilirim (unitary)

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11.1.2 MMSE Signal Detection

$$
\begin{aligned}
E\left\{\|\tilde{\mathbf{z}}_{MMSE}\|_2^2\right\} &= E\left\{\left\|\left(\boldsymbol{\Sigma}+\sigma_z^2\boldsymbol{\Sigma}^{-1}\right)^{-1}\mathbf{U}^H\mathbf{z}\right\|^2\right\} \\
&= E\left\{\mathrm{tr}\left(\left(\boldsymbol{\Sigma}+\sigma_z^2\boldsymbol{\Sigma}^{-1}\right)^{-1}\mathbf{U}^H\mathbf{z}\mathbf{z}^H\mathbf{U}\left(\boldsymbol{\Sigma}+\sigma_z^2\boldsymbol{\Sigma}^{-1}\right)^{-1}\right)\right\} \\
&= \mathrm{tr}\left(\left(\boldsymbol{\Sigma}+\sigma_z^2\boldsymbol{\Sigma}^{-1}\right)^{-1}\mathbf{U}^H E\left\{\mathbf{z}\mathbf{z}^H\right\}\mathbf{U}\left(\boldsymbol{\Sigma}+\sigma_z^2\boldsymbol{\Sigma}^{-1}\right)^{-1}\right) \\
&= \mathrm{tr}\left(\sigma_z^2\left(\boldsymbol{\Sigma}+\sigma_z^2\boldsymbol{\Sigma}^{-1}\right)^{-2}\right) \\
&= \sum_{i=1}^{N_T}\sigma_z^2\left(\sigma_i+\frac{\sigma_z^2}{\sigma_i}\right)^{-2} \\
&= \sum_{i=1}^{N_T}\frac{\sigma_z^2\sigma_i^2}{\left(\sigma_i^2+\sigma_z^2\right)^2}.
\end{aligned}
\qquad (11.12)
$$

hermition

diagonal $\sigma_z^2 I_{N_T \times N_T}$

diag

$U^H U = I$

trace

Sonuç

çok küçük bile olsa sonucu çok etkilemez.

11.1.2 MMSE Signal Detection

$\sigma_{min}^2 = \min\{\sigma_1^2, \sigma_2^2, \cdots, \sigma_{N_T}^2\}$

$$E\left\{\|\tilde{\mathbf{z}}_{ZF}\|_2^2\right\} = \sum_{i=1}^{N_T} \frac{\sigma_z^2}{\sigma_i^2} \approx \frac{\sigma_z^2}{\sigma_{\min}^2} \quad \text{for ZF} \tag{11.13a}$$

$$E\left\{\|\tilde{\mathbf{z}}_{MMSE}\|_2^2\right\} = \sum_{i=1}^{N_T} \frac{\sigma_z^2 \sigma_i^2}{\left(\sigma_i^2 + \sigma_z^2\right)^2} \approx \frac{\sigma_z^2 \sigma_{\min}^2}{\left(\sigma_{\min}^2 + \sigma_z^2\right)^2} \quad \text{for MMSE} \tag{11.13b}$$

gürültü daha az etki eder.

N_T antenten N_T sembol iletilirse, $N_R \geq N_T$ olmak zorunda.

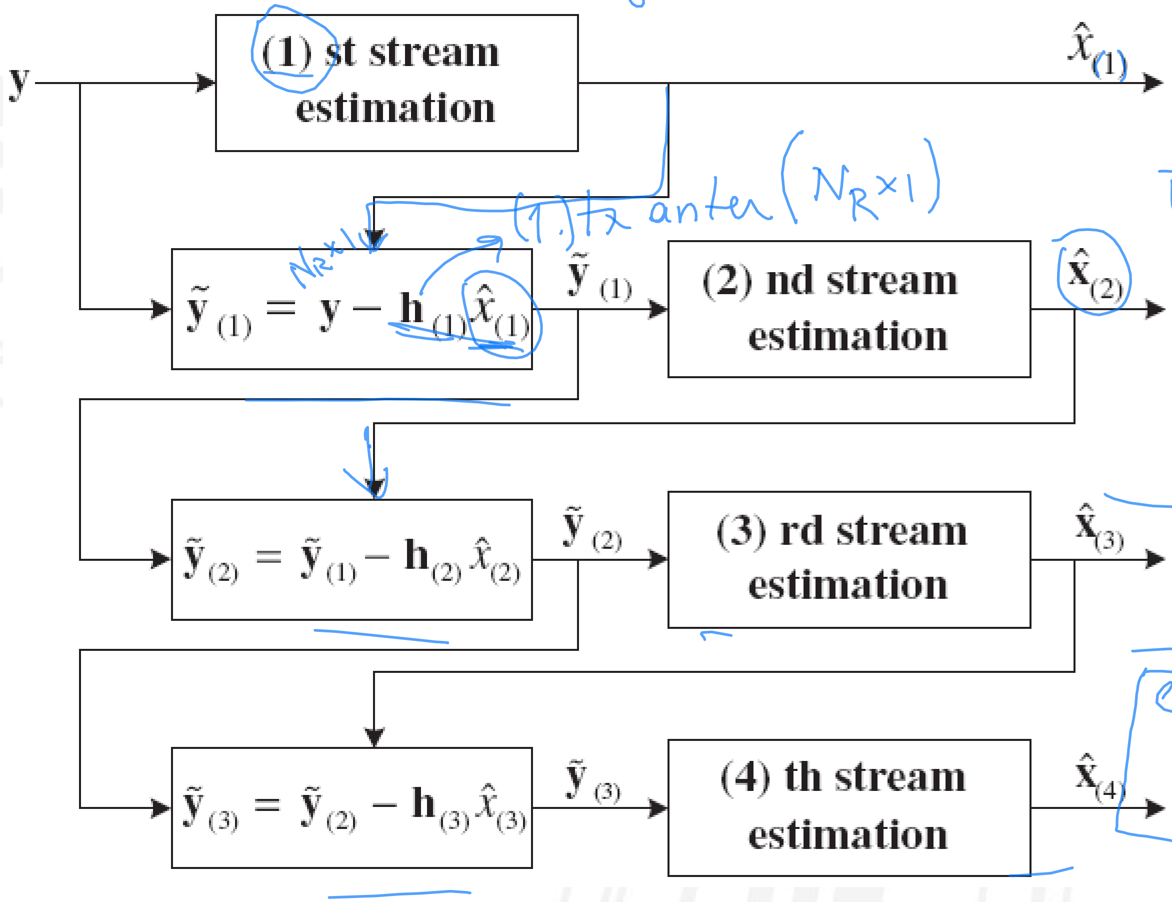
diversity order $N_R - N_T + 1$

$N_T = 1$ ise, ZF MRC ile eşdeğer olur. (diversity order N_R)

Ordered Successive Interference Cancellation

11.2 OSIC Signal Detection

(i) = i olmak zorunda değil.



Bir sembol demodüle edilir.

Toplam sinyalden silinir

2. sembol demod. edilir

ZF veya MMSE uygulanabilir.

error propagation olabilir.

Figure 11.2 Illustration of OSIC signal detection for four spatial streams (i.e., $N_T = 4$).

11.2 OSIC Signal Detection

y

$$\tilde{\mathbf{y}}_{(1)} = \mathbf{y} - \mathbf{h}_{(1)}\hat{x}_{(1)} = \mathbf{h}_{(1)}\left(x_{(1)} - \hat{x}_{(1)}\right) + \mathbf{h}_{(2)}x_{(2)} + \cdots + \mathbf{h}_{(N_T)}x_{(N_T)} + \mathbf{z}. \quad (11.14)$$

Method 1 — 0 — ≠0 ise hata yayılımı (sıra önemli)

MMSE

post detection — i'inci sütun,

SINR

$$\mathrm{SINR}_i = \frac{\mathsf{E}_{\mathsf{x}}\left|\mathbf{w}_{i,MMSE}\mathbf{h}_i\right|^2}{\mathsf{E}_{\mathsf{x}}\sum_{l \neq i}\left|\mathbf{w}_{i,MMSE}\mathbf{h}_l\right| + \sigma_z^2\left\|\mathbf{w}_{i,MMSE}\right\|^2}, \quad i = 1, 2, \ldots, N_T \quad (11.15)$$

diğer sinyaller — gürültü

$N_R \times (N_T - 1)$ oldu.

Method 2

ZF

$$\mathbf{H}^{(1)} = [\mathbf{h}_1\ \mathbf{h}_2\ \cdots\ \mathbf{h}_{l-1}\ \mathbf{h}_{l+1}\ \cdots\ \mathbf{h}_{N_T}] \quad (11.16)$$

h_l'yi çıkarttık

SNR based ordering

$$\mathrm{SNR}_i = \frac{\mathsf{E}_{\mathsf{x}}}{\sigma_z^2\|\mathbf{w}_i\|^2}, \quad i = 1, 2, \ldots, N_T. \quad (11.17)$$

Method 3

Column norm based ordering

$\|h_i\|^2$

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z} = \mathbf{h}_1x_1 + \mathbf{h}_2x_2 + \cdots + \mathbf{h}_{N_T}x_{N_T} + \mathbf{z} \quad (11.18)$$

11.2 OSIC Signal Detection

OSIC diversity order daha fazla $N_R - N_T + i$ inci detect edilen sembolün çeşitlilik derecesi

$N_T = N_R = 4$
4x4 MIMO

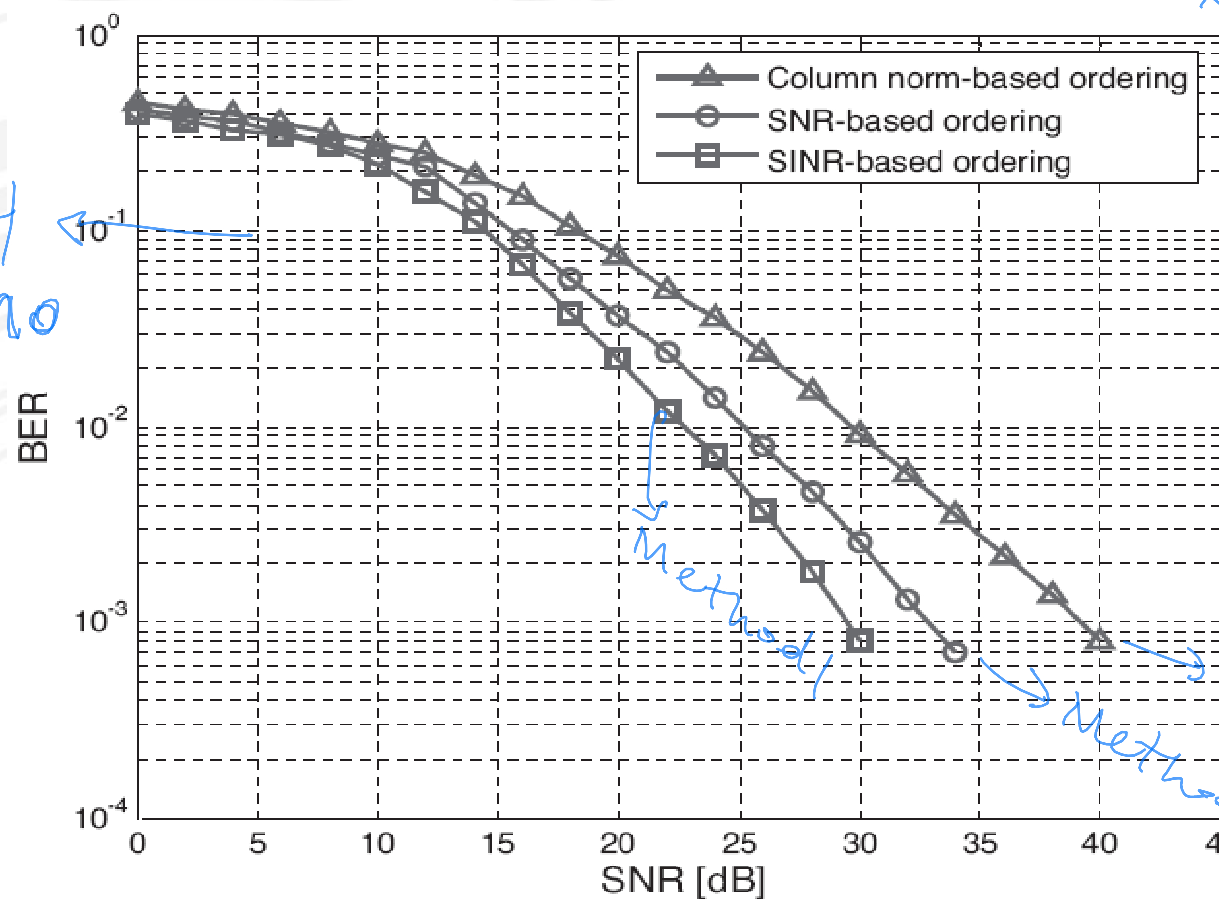


Figure 11.3 Performance of OSIC methods with different detection ordering.

Programlar

- Program 11.1 “OSIC_detector” implementing the various OSIC signal detection methods
- Program 11.2 “QAM16_slicer”
- Program 11.3 “ML_detector” for ML signal detection

11.3 ML Signal Detection

üstel karmaşıklığa sahip.

- Alınan sinyal vektörü (**y**)
- Gönderilmesi olası bütün sinyal vektörleri $\boldsymbol{x} \in C^{N_T}$ için kanal matrisi ile çarpım. $\boldsymbol{Hx}$

16QAM için 16^{N_T} tane şey denenir.

teker teker

- Bunlar arasındaki uzaklığı minimize eden $\boldsymbol{x}$ değeri.

$N_T \times 1$

$$\hat{\mathbf{x}}_{ML} = \underset{\mathbf{x} \in \mathbf{C}^{N_T}}{\operatorname{argmin}} \|\mathbf{y} - \mathbf{Hx}\|^2 \quad (11.19)$$

minimum distance

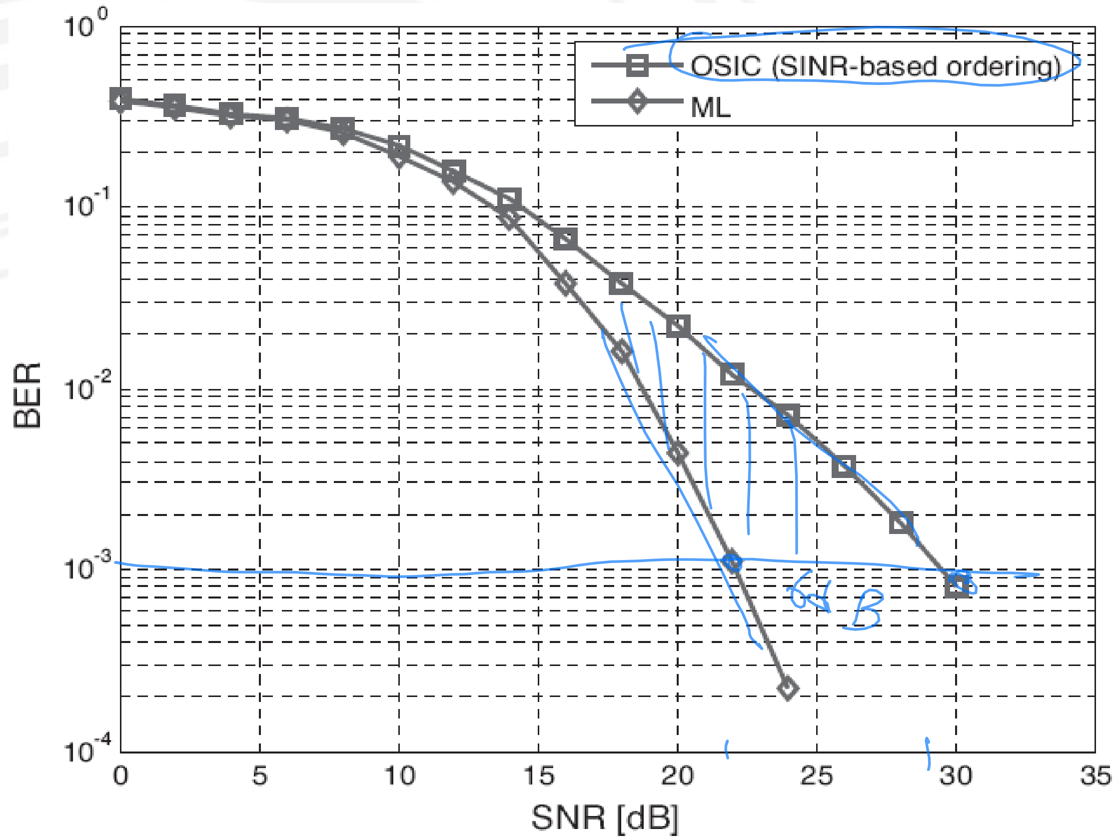
- Bütün vektörler eşit olasılıklı ise bu yöntem optimaldir.
- Bit/sembol (k) ve verici anten sayısı arttıkça karmaşıklık üstel artar $O(2^{kN_T})$

$\log_2 M$

uzun süren → benchmark.

11.3 ML Signal Detection

$$\hat{\mathbf{x}}_{ML} = \underset{\mathbf{x} \in C^{N_T}}{\operatorname{argmin}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 \quad (11.19)$$



Sanırım 4x4 MIMO 16QAM için

Figure 11.4 Performance comparison: OSIC vs. ML detection methods.