Chapter 10: Antenna Diversity and Space –Time Coding Techniques

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10.1 Antenna Diversity

- İki ana yöntem
  - Çeşitleme (Diversity) : Aynı bilginin farklı kanallardan gönderilmesi
  - Çoklama (Multiplexing) Her kanaldan farklı bilgi gönderilmesi
- Anten Çeşitlemesi
  - Sönümlü kanalın değişkenliğine karşı hata performansının kötüleşmesini önler

  - Uzamsal Çeşitleme (Space) Çoklu anten
  - Polarizasyon çeşitlemesi : Yatay ~ dikey
  - Zaman çeşitlemesi Aynı bilgiyi farklı zamanlarda iletmek
  - Frekans çeşitlemesi " " " frekanslarda (ör. OFDM)
  - Açı çeşitlemesi " " farklı yönlü antenlerle iletmek

Çeşitleme türleri

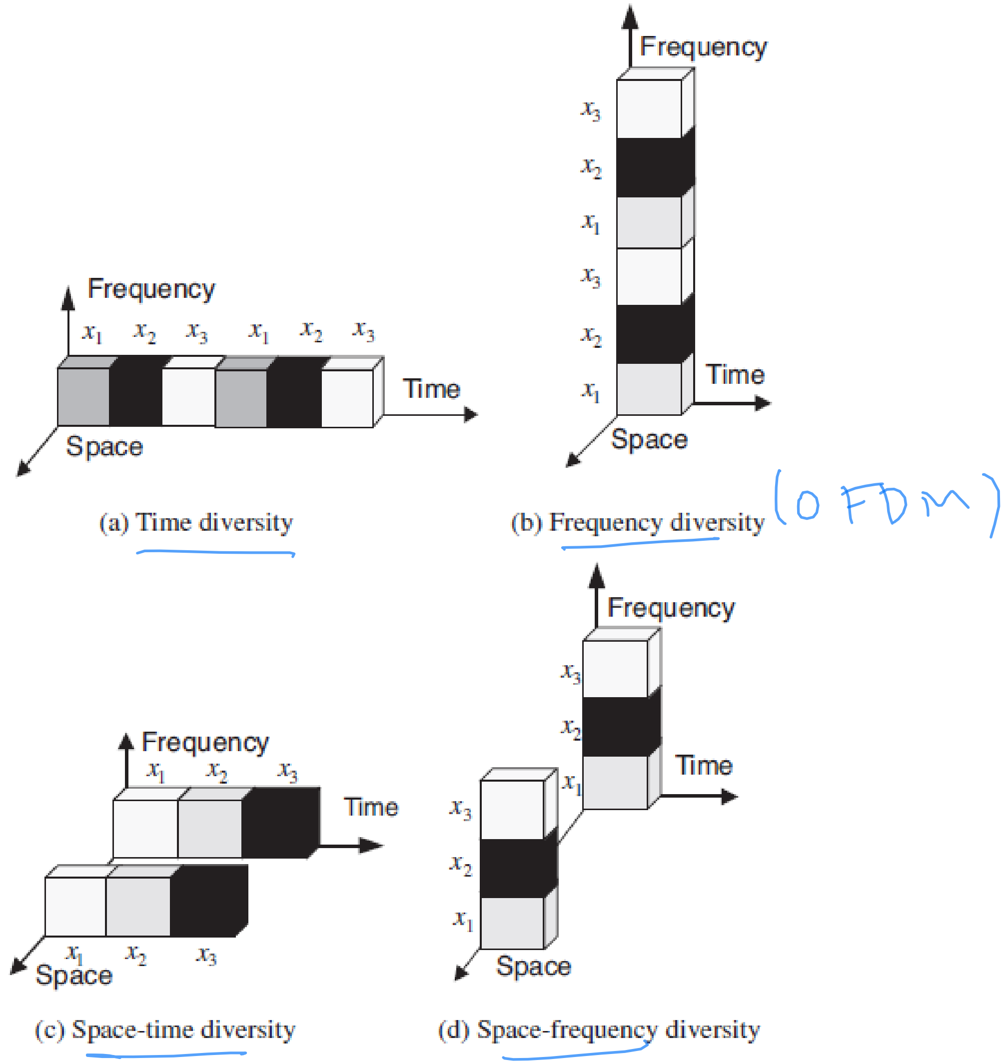
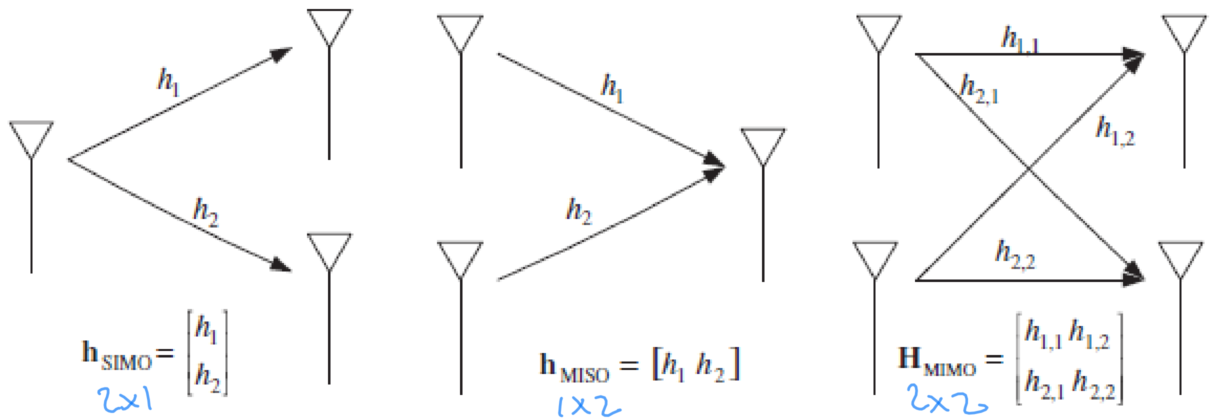


Figure 10.1 Illustration of time, frequency, and space diversity techniques.

Anten konfigürasyonları

$N_R \times N_T$ kanal matrisi



(a) SIMO (b) MISO (c) MIMO

1x2 MIMO 2x1 MIMO 2x2 MIMO

Figure 10.2 Examples of various antenna configurations.

10.1.1. Alıcıda Çeşitleme (Receive Diversity)

(SIMO)

$$\boxed{E[|z|^2]=1}$$

$$\mathbf{h} = \underbrace{[h_1 h_2 \cdots h_{N_R}]}^T \tag{10.1}$$

$$\bar{y} = \sqrt{\frac{E_x}{N_0}}\left[h_1 x + z_1, h_2 x + z_2, \ldots, h_{N_R} x + z_{N_R}\right]$$

$$\mathbf{y} = \sqrt{\frac{\mathsf{E_x}}{\mathsf{N_0}}}\mathbf{h}x + \mathbf{z} \tag{10.2}$$

$\rightarrow$ bir sembol

- Selection (SC) Combining $\rightarrow \underset{i}{\operatorname{argmax}} \left\{|h_i|^2 \frac{E_x}{N_0}\right\}$

$$\gamma_i = |h_i|^2 \frac{\mathsf{E_x}}{\mathsf{N_0}}, \quad i = 1, 2, \ldots, N_R. \tag{10.3}$$

hangi antene gelen sinyal daha yüksek güçlü.

- Maximal Ratio (MC) Combining

$$y_{MRC} = \underbrace{\left[w_1^{(MRC)} w_2^{(MRC)} \cdots w_{N_R}^{(MRC)}\right]}_{\mathbf{w}_{MRC}^T} \mathbf{y} = \boxed{\sum_{i=1}^{N_R} w_i^{(MRC)} y_i} \tag{10.5}$$

$\rightarrow$ post processing

$\bar{w} = ?$

$$\begin{aligned} y_{MRC} &= \mathbf{w}_{MRC}^T \left(\underbrace{\sqrt{\frac{\mathsf{E_x}}{\mathsf{N_0}}}\mathbf{h}x + \mathbf{z}}_{\bar{y}}\right) \\ &= \underbrace{\sqrt{\frac{\mathsf{E_x}}{\mathsf{N_0}}}\mathbf{w}_{MRC}^T \mathbf{h}x}_{\text{bilgi}} + \underbrace{\mathbf{w}_{MRC}^T \mathbf{z}}_{\text{gürültü}} \end{aligned} \tag{10.6}$$

signal : $P_s = E\left\{ \left| \sqrt{\frac{E_x}{N_0}} \, \bar{w}_{MRC}^T \, \bar{h} \, x \right|^2 \right\} = \frac{E_x}{N_0} \left| \bar{w}_{MRC}^T \, \bar{h} \right|^2$

gürültü : $P_z = E\left\{ \left| \bar{w}_{MRC}^T \bar{z} \right|^2 \right\} = \left\| \bar{w}_{MRC}^T \right\|_2^2$

$$\rho_{MRC} = \frac{P_s}{P_z} = \frac{E_x}{N_0} \frac{\left| \bar{w}_{MRC}^T \bar{h} \right|^2}{\left\| \bar{w}_{MRC}^T \right\|_2^2} \underset{\text{Cauchy Schwarz}}{\leq} \frac{E_x}{N_0} \frac{\cancel{\left\| \bar{w}_{MRC} \right\|_2^2} \left\| \bar{h} \right\|_2^2}{\cancel{\left\| \bar{w}_{MRC} \right\|_2^2}}$$

eşitliğin sağlanması için $\bar{w}_{MRC} = \bar{h}^*$ olmalıdır.

$$\bar{w}_{MRC} = \left[h_1^* \; h_2^* \; \cdots \; h_{N_R}^* \right]$$

$$\rho_{MRC} = \frac{E_x}{N_0} \left\| \bar{h} \right\|_2^2 = \frac{E_x}{N_0} \sum_{i=1}^{N_R} \left| h_i \right|^2$$

$$\rho_{SC} = \frac{E_x}{N_0} \max \left| h_i \right|^2$$

Kodlar

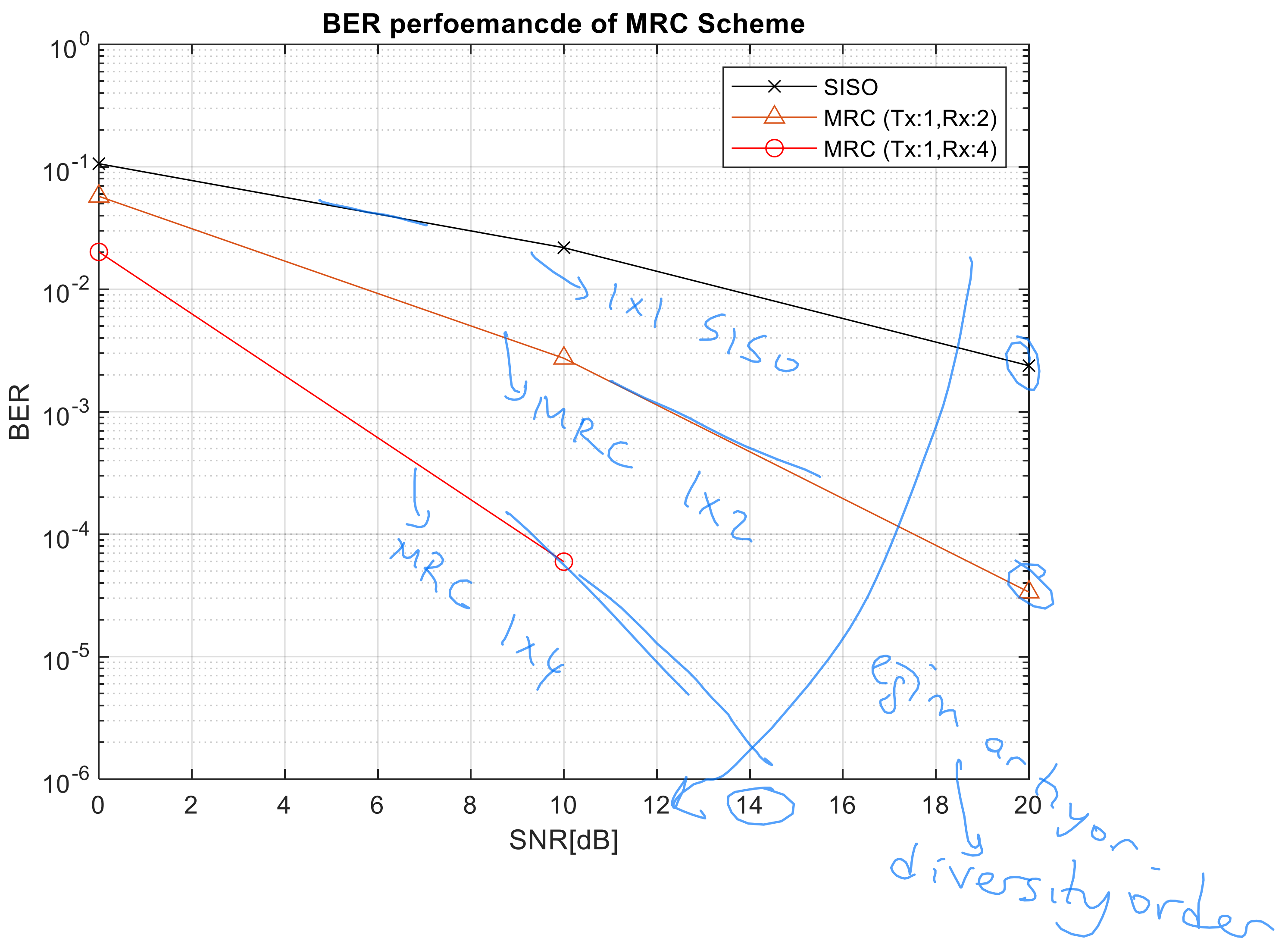
$$y_{EGC} = \frac{E_x}{N_0}\left(\sum_{i=1}^{N_R} h_i\right)x + \sum z_i$$

- Equal Gain Combining (EGC)
  - MRC'nin özel hali. Bütün katsayılar eşit
- MRC en iyi başarımı elde eder.
  - MRC bir tür uzamsal uyumlu filtredir. $\boldsymbol{w}_{MRC} = \boldsymbol{h}^*$

matched filter

- Kodlar
  - Program 10.1 "MRC_scheme.m" for performance of MRC for Rayleigh fading channels
  - Program 10.2 "modulator" for BPSK, QPSK, 8-PSK, 16-QAM mapping function

MRC Performansı



10.2 Uzay Zaman Kodlama (Space-Time Coding)

- Bir verici çeşitlemesi türü (transmit diversity)
- Semboller T zaman boyunca N_T antene eşleniyor.
- T zaman diliminde N sembol , R= $\frac{N}{T} \leq 1$
- Alıcıda semboller N_R antene ulaşıyor

anten

timeslot

$$y_j^{(t)} = \sqrt{\frac{\mathsf{E_x}}{\mathsf{N_0}N_T}}\left[h_{j1}^{(t)}\ h_{j2}^{(t)} \cdots h_{jN_T}^{(t)}\right]\underbrace{\begin{bmatrix} x_1^{(t)} \\ x_2^{(t)} \\ \vdots \\ x_{N_T}^{(t)} \end{bmatrix}}_{\mathbf{x}^{(t)}} + z_j^{(t)} \qquad (10.13)$$

rx anten

zaman dilimi

$\boxed{N_T \cdot T}$

elinde N_T T slot var bu slotları kullanarak $N \leq T$ adet sembolü mini hata ile göndermek.

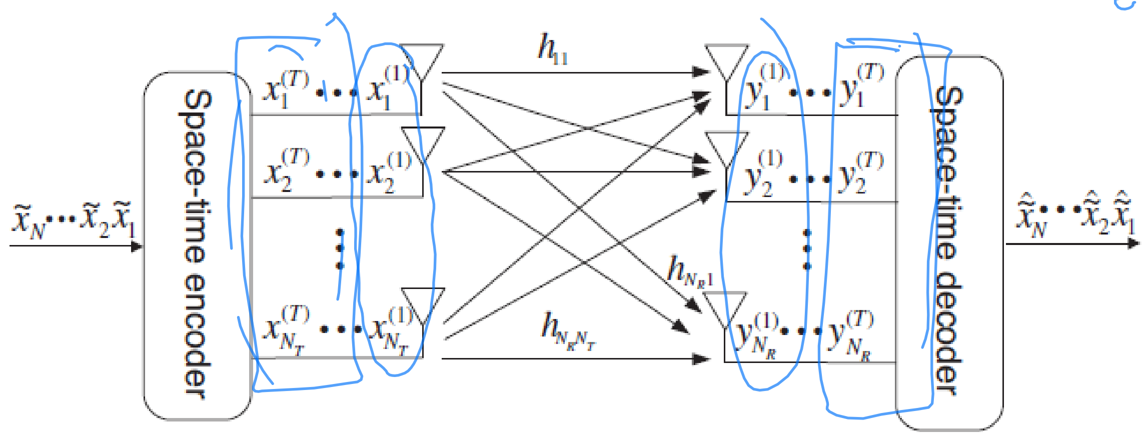


Figure 10.4 Space-time coded MIMO systems.

10.2.2 Pairwise Error Probability

$\bar{X}_{ML} \neq \bar{X}$ olasılığı

mükemmel CSIR varsayımı (sf 289-291)
analiz

Q func.

$$\Pr(\mathbf{X} \to \mathbf{X}_{ML}) = Q\left(\sqrt{\frac{\mathsf{E_x}}{2\mathsf{N_0}N_T}}\|\mathbf{H}(\mathbf{X}-\mathbf{X}_{ML})\|_F\right) \tag{10.28}$$

chernoff bound $Q(x) \leq \frac{1}{2}e^{-x^2/2}$

$$\Pr(\mathbf{X} \to \mathbf{X}_{ML}) \leq \frac{1}{2}\exp\left(-\frac{\mathsf{E_x}}{\mathsf{N_0}N_T}\frac{\|\mathbf{H}(\mathbf{X}-\mathbf{X}_{ML})\|_F^2}{4}\right) \tag{10.29}$$

$$\bar{X} = \begin{bmatrix} x_1^{(1)} & \cdots & x_1^{(T)} \\ \vdots & \ddots & \vdots \\ x_{N_T}^{(1)} & \cdots & x_{N_T}^{(T)} \end{bmatrix} \qquad \bar{H} = \begin{bmatrix} h_{11} & \cdots & h_{1N_T} \\ \vdots & & \vdots \\ h_{N_R 1} & \cdots & h_{N_R N_T} \end{bmatrix}$$

$N_T \times T$ matris $\qquad$ $N_R \times N_T$ matris

10.2.3 Uzay Zaman Kod Tasarımı

$$
\begin{aligned}
\|\underbrace{\mathbf{H}}_{N_R \times N_T}(\underbrace{\mathbf{X}-\hat{\mathbf{X}}}_{N_T \times T})\|_F^2 &= \operatorname{tr}\left(\mathbf{H}(\mathbf{X}-\hat{\mathbf{X}})(\mathbf{X}-\hat{\mathbf{X}})^H\mathbf{H}^H\right) \\
&= \sum_{l=1}^{N_R} \mathbf{h}_l(\mathbf{X}-\hat{\mathbf{X}})(\mathbf{X}-\hat{\mathbf{X}})^H\mathbf{h}_l^H
\end{aligned}
\tag{10.30}
$$

$\to$ H'nin l'inci satırı $\to 1 \times N_T$

$$
\sum_{l=1}^{N_R} \mathbf{h}_l \underbrace{(\mathbf{X}-\hat{\mathbf{X}})(\mathbf{X}-\hat{\mathbf{X}})^H}_{V \Lambda V^H} \mathbf{h}_l^H = \sum_{l=1}^{N_R} \mathbf{h}_l \mathbf{V}\Lambda\mathbf{V}^H\mathbf{h}_l^H
\tag{10.31}
$$

$V \to$ unitary

$[\bar{v}_1 \; \bar{v}_2 \; \cdots \; \bar{v}_{N_T}] \to$ sütunlar

$\Lambda \to \operatorname{diag}(\lambda_1, \lambda_2, \ldots, \lambda_{N_T}) \to (\bar{X}-\hat{\bar{X}})(\bar{X}-\hat{\bar{X}})^H$ özdeğerleri

$$
\sum_{l=1}^{N_R} h_l V \Lambda V^H h_l^H = \underbrace{\sum_{l=1}^{N_R} \sum_{i=1}^{N_T} \lambda_i \underbrace{|\bar{h}_l \bar{V}_i|^2}_{\beta_{l,i}}} \to \max
$$

$$\Pr(X \to \hat{X} \mid H) \le \frac{1}{2} \exp\left(-\frac{E_x}{4N_0 N_T} \sum_{l=1}^{N_R} \sum_{i=1}^{N_T} \lambda_i \left| \beta_{l,i} \right|^2 \right)$$

$\vdots$

$$\Pr(X \to \hat{X} \mid H) \le \left(\prod_{i=1}^{N_T} \frac{1}{1 + \frac{E_x}{4N_0} \lambda_i} \right)^{N_R} \le \left(\prod_{i=1}^{r} \left(\frac{E_x}{4N_0} \lambda_i \right)^{-1} \right)^{N_R} = \underline{\left(\prod^{r} \lambda_i \right)^{-N_R} \left(\frac{E_x}{4N_0} \right)^{-rN_R}}$$

$$r = \text{rank}\left((X - \hat{X})(X - \hat{X})^H \right)$$

$$\log\left(\Pr(X \to \hat{X}) \right) \le -N_R \log\left(\prod \lambda_i \right) - rN_R \log\left(\frac{E_x}{4N_0} \right)$$

$\boxed{\max\ rN_R}$

$\boxed{\max \sum_{i=1}^{r} \lambda_i}$ ☆

- Diversity order: NR*v<NR*NT

$$v = \min_{p \neq q} \text{rank}\{(\mathbf{X}_p - \mathbf{X}_q)(\mathbf{X}_p - \mathbf{X}_q)^H\} \tag{10.39}$$

$N_r \cdot v =$ diversity order.

$T \ge N_T \Rightarrow \max(v) = N_T$

$N_R N_T$ max diver. order.

- Coding gain

$$\Lambda_{\min} = \min_{p \neq q} \prod_{i=1}^{N_T} \lambda_i(\mathbf{X}_p, \mathbf{X}_q). \tag{10.41}$$

Kodlama ve Çeşitleme Kazancı

- Çeşitleme kazancı: loglog BER grafiğinin eğimi
  - $T \geq N_T$ ise maksimum $N_R N_T$ olabilir. (rank kriteri)
- Kodlama kazancı: loglog BER grafiğinin sola kayması

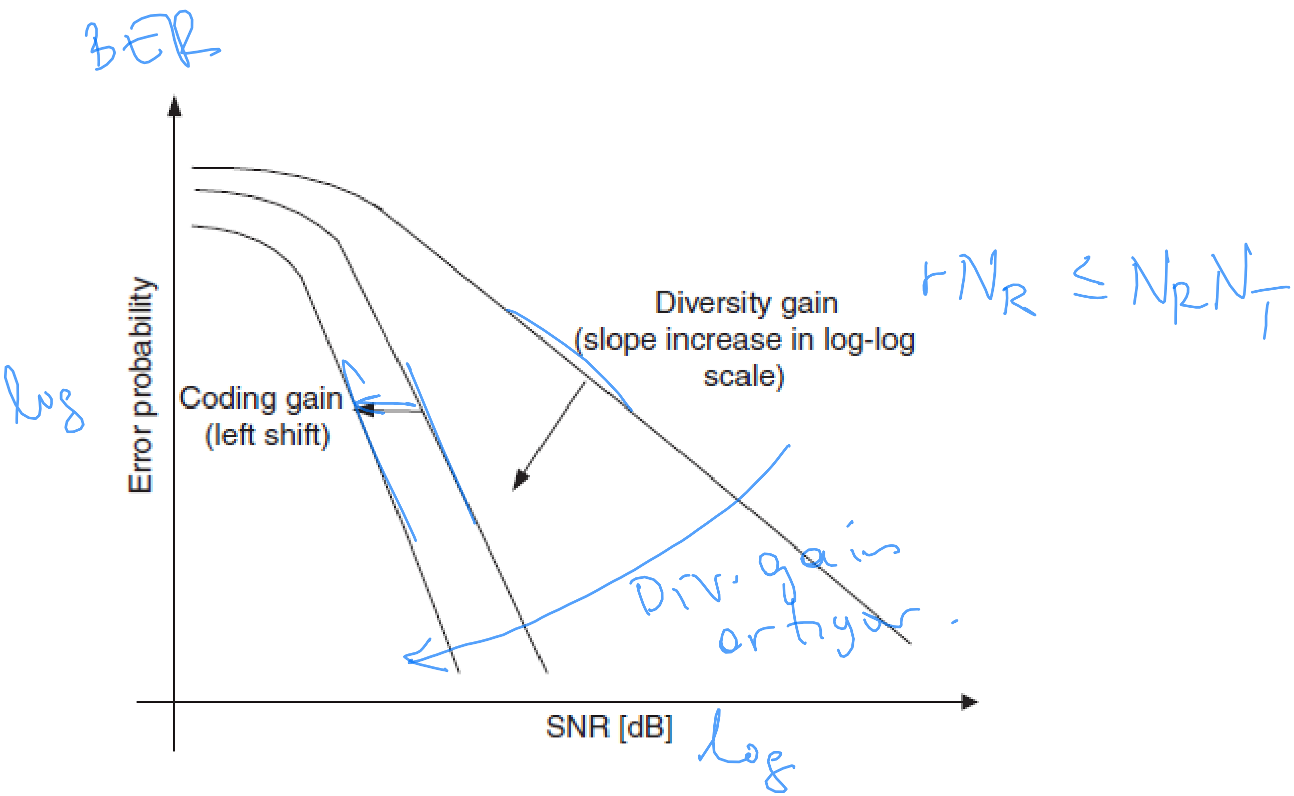


Figure 10.5 Diversity gain vs. coding gain.

10.3 Uzay Zaman Blok Kodu

10.3.1 Alamouti Kodu

2x1 MIMO için (2tx − 1rx)

$$\begin{array}{ccc} & t=1 & t=2 \\ \text{ant 1} \\ \text{ant 2} & \mathbf{X} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \end{array} \qquad (10.42)$$

$$\boxed{R = \frac{2}{2} = 1}$$

SISO gibi ama çok daha düşük BER

- $N_T = 2, T = 2, N = 2$
- Kod oranı 1

x_1, x_2

$$\bar{X}\bar{X}^H = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \begin{bmatrix} x_1^* & x_2^* \\ -x_2 & x_1 \end{bmatrix} = \begin{bmatrix} |x_1|^2+|x_2|^2 & 0 \\ 0 & |x_1|^2+|x_2|^2 \end{bmatrix}$$

→ ortogonal

→ full rank.

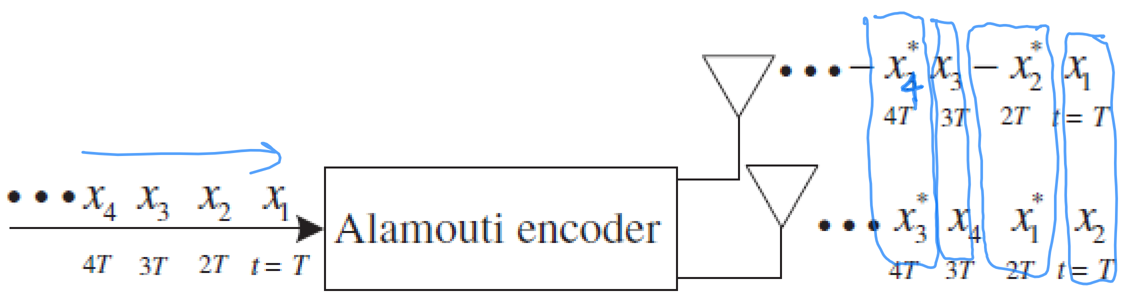


Figure 10.6 Alamouti encoder.

$$\bar{X}_p = \begin{bmatrix} x_{1,p} & -x_{2,p}^* \\ x_{2,p} & x_{1,p}^* \end{bmatrix}, \quad \bar{X}_q = \begin{bmatrix} x_{1,q} & -x_{2,q}^* \\ x_{2,q} & x_{1,q}^* \end{bmatrix}$$

(10.39) $$(\bar{X}_p - \bar{X}_q)(\bar{X}_p - \bar{X}_q)^H = \begin{bmatrix} x_{1,p} - x_{1,q} & x_{2,q}^* - x_{2,p}^* \\ x_{2,p} - x_{2,q} & x_{1,p}^* - x_{1,q}^* \end{bmatrix} \times \begin{bmatrix} x_{1,p}^* - x_{1,q}^* & x_{2,p}^* - x_{2,q}^* \\ x_{2,q} - x_{2,p} & x_{1,p} - x_{1,q} \end{bmatrix}$$

$e_1 = x_{1,p} - x_{1,q}$

$e_2 = x_{2,p} - x_{2,q}$

$$\begin{bmatrix} e_1 & -e_2^* \\ e_2 & e_1^* \end{bmatrix} \circ \begin{bmatrix} e_1^* & e_2^* \\ -e_2 & e_1 \end{bmatrix}$$

$$= \begin{bmatrix} |e_1|^2 + |e_2|^2 & 0 \\ 0 & |e_1|^2 + |e_2|^2 \end{bmatrix} \longrightarrow \text{rank} = 2$$

çünkü e_1 ve e_2 aynı anda 0 olamaz ($\bar{X}_p \neq X_q$)

diversity gain 2 $= \underset{1}{N_R}\,\overset{2}{N_T}$

10.3.1 Alamouti Alıcısı (Maximum Likelihood)

- Kanalın, iki sembol süresi boyunca sabit kaldığını varsayıyoruz.

$$t=1 \qquad y_1 = h_1x_1 + h_2x_2 + z_1$$

$$t=2 \qquad y_2 = -h_1x_2^* + h_2x_1^* + z_2 \qquad (10.47)$$

$$\begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

Varsayım

$$h_1(t) = h_1(t+T_s)$$

$$h_2(t) = h_2(t+T_s)$$

$$\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{bmatrix} = \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} =$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_1x_1 + h_2x_2 + z_1 \\ -h_1x_2^* + h_2x_1^* + z_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_1x_1 + h_2x_2 + z_1 \\ -h_1^*x_2 + h_2^*x_1 + z_2^* \end{bmatrix}$$

$$\begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} \begin{bmatrix} h_1 & h_2 \\ -h_1^* & h_2^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

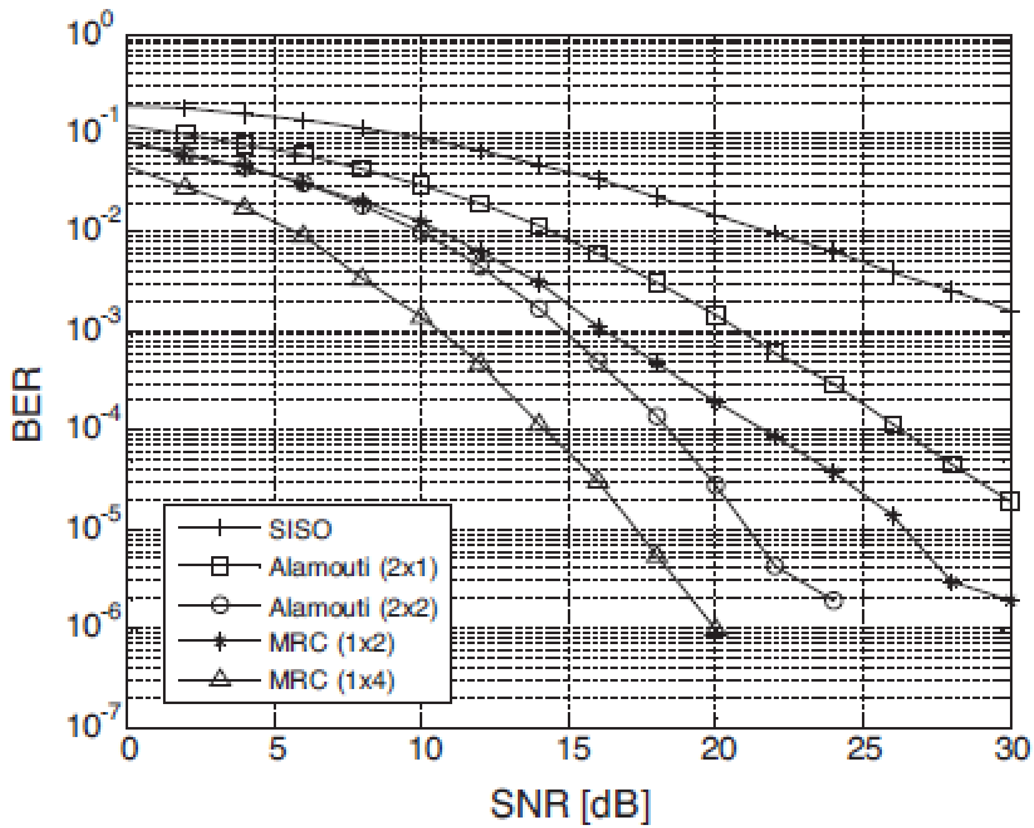
$$\tilde{y}_1 = (|h_1|^2 + |h_2|^2)\, x_1 + \tilde{z}_1$$

$$\tilde{y}_2 = (|h_1|^2 + |h_2|^2)\, x_2 + \tilde{z}_2$$

$$\hat{x}_{i,ML} = Q\left(\frac{\tilde{y}_i}{|h_1|^2 + |h_2|^2}\right), \quad i = 1, 2. \qquad (10.52)$$

qamdemod

Performans



- Program 10.3 “Alamouti_scheme.m” for Alamouti space-time block coding
- MRC 1x2 ile Alamouti 2x1 aynı çeşitleme kazancı
- 2x2 Alamouti ile MRC 1x4 aynı çeşitleme kazancı

$N_T \geq 2$

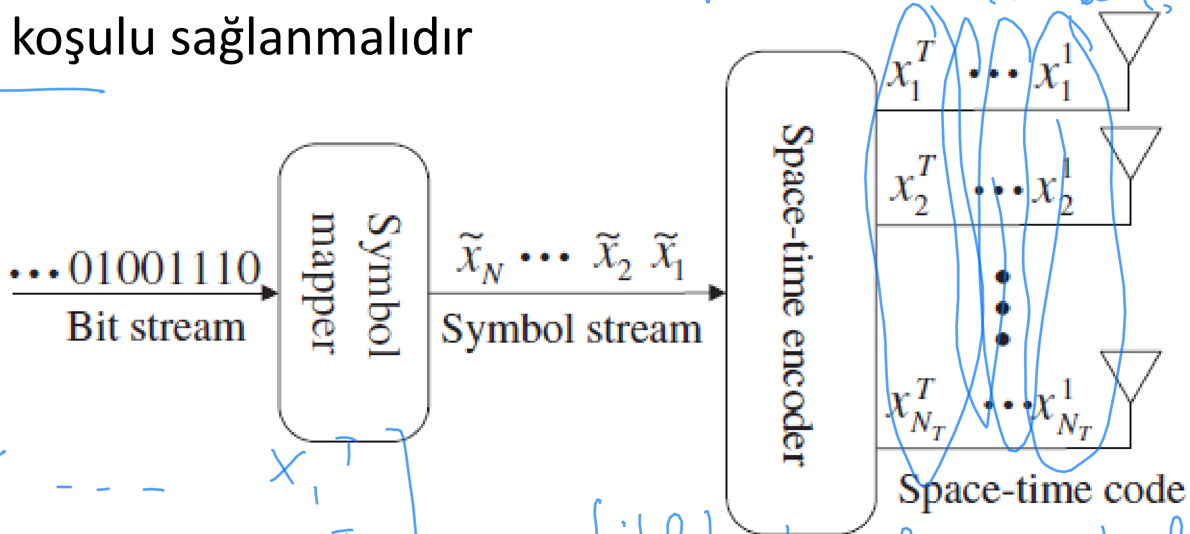
10.3.2 Genel Uzay Zaman Kodları

$$\mathbf{X}\mathbf{X}^H = c\left(|x_i^1|^2 + |x_i^2|^2 + \cdots + |x_i^T|^2\right)\mathbf{I}_{N_T} \qquad (10.53)$$
$$= c\|\mathbf{x}_i\|^2 \mathbf{I}_{N_T}$$

$N_T \times T$

diagonal, full rank olmalı

- Diklik koşulu sağlanmalıdır



$$\begin{bmatrix} x_1^1 & x_1^2 & \cdots & x_1^T \\ x_2^1 & x_2^2 & \cdots & x_2^T \\ \vdots & & \ddots & \vdots \\ x_{N_T}^1 & x_{N_T}^2 & \cdots & x_{N_T}^T \end{bmatrix}$$

$N_T \times T$ matris

diklik koşulu: satırları dik olmalı

$$\bar{x}_i\, \bar{x}_j^H = 0$$
$$= \sum_{t=1}^{T} x_i^t \left(x_j^t\right)^* = 0 \quad \forall i \neq j$$

10.3.2.1 Gerçel Uzak Zaman Blok Kodları

Bütün semboller gerçel ise (PAM, BPSK)

$N=2$ x_1, x_2

$T=2$

$t=1$ $t=2$

$$\mathbf{X}_{2,\text{real}} = \begin{bmatrix} x_1 & -x_2 \\ x_2 & x_1 \end{bmatrix} \tag{10.55}$$

$x_1 x_2^* - x_2 x_1^* = 0$

$= x_1 x_2 - x_2 x_1 = 0$

x_i ler gerçel ise.

$N=4$ x_1, x_2, x_3, x_4

$T=4$

$$\mathbf{X}_{4,\text{real}} = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 \\ x_2 & x_1 & x_4 & -x_3 \\ x_3 & -x_4 & x_1 & x_2 \\ x_4 & x_3 & -x_2 & x_1 \end{bmatrix} \tag{10.56}$$

$\cancel{x_1 x_2 - x_2 x_1} - \cancel{x_3 x_4 + x_3 x_4} = 0$

kod oranı: $\dfrac{N}{T} = 1$ mümkün.

$N=8$

$T=8$

$$\mathbf{X}_{8,\text{real}} = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & -x_5 & -x_6 & -x_7 & -x_8 \\ x_2 & x_1 & -x_4 & x_3 & -x_6 & x_5 & x_8 & -x_7 \\ x_3 & x_4 & x_1 & -x_2 & -x_7 & -x_8 & x_5 & x_6 \\ x_4 & -x_3 & x_2 & x_1 & -x_8 & x_7 & -x_6 & x_5 \\ x_5 & x_6 & x_7 & x_8 & x_1 & -x_2 & -x_3 & -x_4 \\ x_6 & -x_5 & x_8 & -x_7 & x_2 & x_1 & x_4 & -x_3 \\ x_7 & -x_8 & -x_5 & x_6 & x_3 & -x_4 & x_1 & x_2 \\ x_8 & x_7 & -x_6 & -x_5 & x_4 & x_3 & -x_2 & x_1 \end{bmatrix} \tag{10.57}$$

Semboller karmaşık olsaydı (QAM, PSK) diklik şartı sağlanmazdı.

10.3.2.1 Gerçel Uzak Zaman Blok Kodları

$R = \frac{4}{4}$ $\begin{cases} N_T = 3 \\ T = 4 \end{cases}$ $N = 4$

$$\underline{\mathbf{x}_{3,\text{real}}} = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 \\ x_2 & x_1 & x_4 & -x_3 \\ x_3 & -x_4 & x_1 & x_2 \end{bmatrix} \quad (10.59)$$

$N = 8$

$N_T = 5$

$T = 8$

$\boxed{R = 1}$

$$\mathbf{x}_{5,\text{real}} = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & -x_5 & -x_6 & -x_7 & -x_8 \\ x_2 & x_1 & -x_4 & x_3 & -x_6 & x_5 & x_8 & -x_7 \\ x_3 & x_4 & x_1 & -x_2 & -x_7 & -x_8 & x_5 & x_6 \\ x_4 & -x_3 & x_2 & x_1 & -x_8 & x_7 & -x_6 & x_5 \\ x_5 & x_6 & x_7 & x_8 & x_1 & -x_2 & -x_3 & -x_4 \end{bmatrix} \quad (10.60)$$

$N = 8$

$N_T = 6$

$T = 8$

$R = 1$

$$\mathbf{x}_{6,\text{real}} = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & -x_5 & -x_6 & -x_7 & -x_8 \\ x_2 & x_1 & -x_4 & x_3 & -x_6 & x_5 & x_8 & -x_7 \\ x_3 & x_4 & x_1 & -x_2 & -x_7 & -x_8 & x_5 & x_6 \\ x_4 & -x_3 & x_2 & x_1 & -x_8 & x_7 & -x_6 & x_5 \\ x_5 & x_6 & x_7 & x_8 & x_1 & -x_2 & -x_3 & -x_4 \\ x_6 & -x_5 & x_8 & -x_7 & x_2 & x_1 & x_4 & -x_3 \end{bmatrix} \quad (10.61)$$

10.3.2.1 Gerçel Uzak Zaman Blok Kodları

$$X_{3,real} = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 \\ x_2 & x_1 & x_4 & -x_3 \\ x_3 & -x_4 & x_1 & x_2 \end{bmatrix}$$

- Çeşitleme kazancı

$p \neq q$

$$X_{3,real,p} - X_{3,real,q} = \begin{bmatrix} x_{1,p}-x_{1,q} & x_{2,q}-x_{2,p} & x_{3,q}-x_{3,p} & x_{4,q}-x_{4,p} \\ x_{2,p}-x_{2,q} & x_{1,p}-x_{1,q} & x_{4,p}-x_{4,q} & x_{3,q}-x_{3,p} \\ x_{3,p}-x_{3,q} & x_{4,q}-x_{4,p} & x_{1,p}-x_{1,q} & x_{2,p}-x_{2,q} \end{bmatrix}$$

$$\boxed{X_{3,p} \neq X_{3,q}}$$

$$= \begin{bmatrix} e_1 & -e_2 & -e_3 & -e_4 \\ e_2 & e_1 & e_4 & -e_3 \\ +e_3 & -e_4 & e_1 & e_2 \end{bmatrix} \begin{bmatrix} e_1 & e_2 & e_3 \\ -e_2 & e_1 & -e_4 \\ -e_3 & e_4 & e_1 \\ -e_4 & -e_3 & e_2 \end{bmatrix}$$

$$= \begin{bmatrix} e_1^2+e_2^2+e_3^2+e_4^2 & 0 & 0 \\ 0 & e_1^2+e_2^2+e_3^2+e_4^2 & 0 \\ 0 & 0 & e_1^2+e_2^2+e_3^2+e_4^2 \end{bmatrix} \rightarrow \text{min rank} = 3$$

10.3.2.2 Karmaşık Uzak Zaman Blok Kodları

ör: AM, PSK

- $N_T \geq 3$ için maksimum Çeşitleme kazancı ve kod oranı beraber sağlanamaz.
- $N_T \geq 2$: Alamouti kodu

$R = \frac{2}{2} = 1$, div. order $= 2$

satırlar dik.

$$\mathbf{X}_{2,\text{complex}} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \quad (10.64)$$

$\cancel{x_1 x_2^*} - \cancel{x_2^* x_1} = 0$

$$\mathbf{X}_{3,\text{complex}} = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & x_1^* & -x_2^* & -x_3^* & -x_4^* \\ x_2 & x_1 & x_4 & -x_3 & x_2^* & x_1^* & x_4^* & -x_3^* \\ x_3 & -x_4 & x_1 & x_2 & x_3^* & -x_4^* & x_1^* & x_2^* \end{bmatrix} \quad (10.65)$$

$N_T = 3$ $N = 4$

$T = 8$

$R = 4/8 = 1/2$

$$\mathbf{X}_{4,\text{complex}} = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & x_1^* & -x_2^* & -x_3^* & -x_4^* \\ x_2 & x_1 & x_4 & -x_3 & x_2^* & x_1^* & x_4^* & -x_3^* \\ x_3 & -x_4 & x_1 & x_2 & x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & x_3 & -x_2 & x_1 & x_4^* & x_3^* & -x_2^* & x_1^* \end{bmatrix} \quad (10.66)$$

$N = 4$

$N_T = 4$

$T = 8$

$\cancel{x_1 x_2^*} - \cancel{x_2 x_1^*} - \cancel{x_3 x_4^*} + \cancel{x_4 x_3^*} + \cancel{x_1^* x_2} - \cancel{x_2^* x_1} - \cancel{x_3^* x_4} + \cancel{x_4^* x_3} = 0$

$R = 1/2$

10.3.2.2 Karmaşık Uzak Zaman Blok Kodları

- Üstteki kodlar ½ oranlı
- Alttakiler daha yüksek oranlı ama daha fazla karmaşıklık var.

$R = 3/4$ $\begin{cases} N_T = 3 \\ N = 3 \\ T = 4 \end{cases}$

$$\mathbf{X}_{3,\text{complex}}^{\text{high rate}} = \begin{bmatrix} x_1 & -x_2^* & \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} \\ x_2 & x_1^* & \frac{x_3^*}{\sqrt{2}} & \frac{-x_3^*}{\sqrt{2}} \\ \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} & \frac{(-x_1 - x_1^* + x_2 - x_2^*)}{2} & \frac{(x_2 + x_2^* + x_1 - x_1^*)}{2} \end{bmatrix} \quad (10.67)$$

$N_T = 4$
$N = 3$
$T = 4$
$\downarrow$
$R = 3/4$

$$\mathbf{X}_{4,\text{complex}}^{\text{high rate}} = \begin{bmatrix} x_1 & -x_2 & \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} \\ x_2 & x_1 & \frac{x_3^*}{\sqrt{2}} & \frac{-x_3^*}{\sqrt{2}} \\ \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} & \frac{(-x_1 - x_1^* + x_2 - x_2^*)}{2} & \frac{(x_2 + x_2^* + x_1 - x_1^*)}{2} \\ \frac{x_3}{\sqrt{2}} & \frac{-x_3}{\sqrt{2}} & \frac{(-x_2 - x_2^* + x_1 - x_1^*)}{2} & -\frac{(x_1 + x_1^* + x_2 - x_2^*)}{2} \end{bmatrix} \quad (10.68)$$

Alıcıda yapılan işler,

10.3.3 Uzay Zaman Kod Çözümü

$N_T = 4$

$T = 4$

$N_R = 1$

$\boxed{X_{4,real}}$

4x4

kanal

t=1,2,3,4

sabit.

t=1 t=2 t=3 t=4

$$[y_1\ y_2\ y_3\ y_4] = \sqrt{\frac{\mathsf{E_x}}{4\mathsf{N_0}}}\underbrace{[h_1\ h_2\ h_3\ h_4]}_{1\times 4}\underbrace{\begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 \\ x_2 & x_1 & x_4 & -x_3 \\ x_3 & -x_4 & x_1 & x_2 \\ x_4 & x_3 & -x_2 & x_1 \end{bmatrix}}_{4\times 4} + [z_1\ z_2\ z_3\ z_4], \qquad (10.69)$$

1x4 MIMO kanal matrisi

noise Var(1)

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}}_{\mathbf{y}_{\text{eff}}} = \sqrt{\frac{\mathsf{E_x}}{4\mathsf{N_0}}}\underbrace{\begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2 & -h_1 & h_4 & -h_3 \\ h_3 & -h_4 & -h_1 & h_2 \\ h_4 & h_3 & -h_2 & -h_1 \end{bmatrix}}_{\mathbf{H}_{\text{eff}}}\underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{\mathbf{x}_{\text{eff}}} + \underbrace{\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}}_{\mathbf{z}_{\text{eff}}} \qquad (10.70)$$

t

$$\tilde{y}_{eff} = \bar{H}_{eff}^T\, y_{eff}$$

$$= \sqrt{\frac{E_x}{4N_0}}\ \underbrace{\bar{H}_{eff}^T H_{eff}}_{\left(\sum_{i=1}^{4}|h_i|^2\right) I_4}\ x_{eff} + \bar{H}_{eff}^T\, \bar{z}_{eff} \longrightarrow x_1, x_2, x_3, x_4 \text{ ayrıştırıldı.}$$

$$\longrightarrow x_{i,ML} = Q\left(\frac{\tilde{y}_{eff}}{\sqrt{\frac{E_x}{4N_0}}\sum |h_i|^2}\right)$$

- Tek alıcı, 4 verici anten için ve gerçel bir STBC için

OFDM

k. subcarrier

$$\sqrt{\frac{E_x}{4N_0}}\,[H_1^k\ H_2^k\ H_3^k\ H_4^k]\begin{bmatrix} X_{4,real} \end{bmatrix} + [Z_1^k\ Z_2^k\ Z_3^k\ Z_4^k]$$

$\bar{H} = fft(h)$

10.3.3 Uzay Zaman Kod Çözümü

QAM, PSK.

$\mathcal{X}_3$, complex

$N_R = 1$

$N_T = 3$

$T = 8$

$N = 4$

$$\underbrace{[y_1\ y_2\ y_3\ y_4 y_5\ y_6\ y_7\ y_8]}_{t} = \sqrt{\frac{E_x}{3N_0}}[h_1\ h_2\ h_3]\begin{array}{c} \qquad\quad t=2 \qquad\qquad\qquad\qquad\qquad \\ \left[\begin{array}{cccc|cccc} x_1 & -x_2 & -x_3 & -x_4 & x_1^* & -x_2^* & -x_3^* & -x_4^* \\ x_2 & x_1 & x_4 & -x_3 & x_2^* & x_1^* & x_4^* & -x_3^* \\ x_3 & -x_4 & x_1 & x_2 & x_3^* & -x_4^* & x_1^* & x_2^* \end{array}\right] \\ \qquad\qquad t=3 \qquad\qquad \xrightarrow{\quad t \quad} \end{array}$$

$$+ [z_1\ z_2\ z_3\ z_4\ z_5\ z_6\ z_7\ z_8] \tag{10.73}$$

$$t = 1:8 \quad \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5^* \\ y_6^* \\ y_7^* \\ y_8^* \end{bmatrix}}_{\mathbf{y}_{\text{eff}}} = \sqrt{\frac{E_x}{3N_0}} \underbrace{\begin{bmatrix} h_1 & h_2 & h_3 & 0 \\ h_2 & -h_1 & 0 & -h_3 \\ h_3 & 0 & -h_1 & h_2 \\ 0 & h_3 & -h_2 & -h_1 \\ h_1^* & h_2^* & h_3^* & 0 \\ h_2^* & -h_1^* & 0 & -h_3^* \\ h_3^* & 0 & -h_1^* & h_2^* \\ 0 & h_3^* & -h_2^* & -h_1^* \end{bmatrix}}_{\mathbf{H}_{\text{eff}}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{\mathbf{x}_{\text{eff}}} + \underbrace{\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5^* \\ z_6^* \\ z_7^* \\ z_8^* \end{bmatrix}}_{\mathbf{z}_{\text{eff}}} \tag{10.74}$$

- Tek alıcı, üç verici anten için ve karmaşık bir STBC için

$$\tilde{y}_{eff} = \bar{H}_{eff}^H y_{eff} \Rightarrow \sqrt{\frac{E_x}{3N_0}}\ \underbrace{\bar{H}_{eff}^H \bar{H}_{eff}}_{2\sum|h_j|\ \bar{I}_3}\ \bar{x}_{eff} + \bar{H}_{eff}^H \bar{z}_{eff}$$

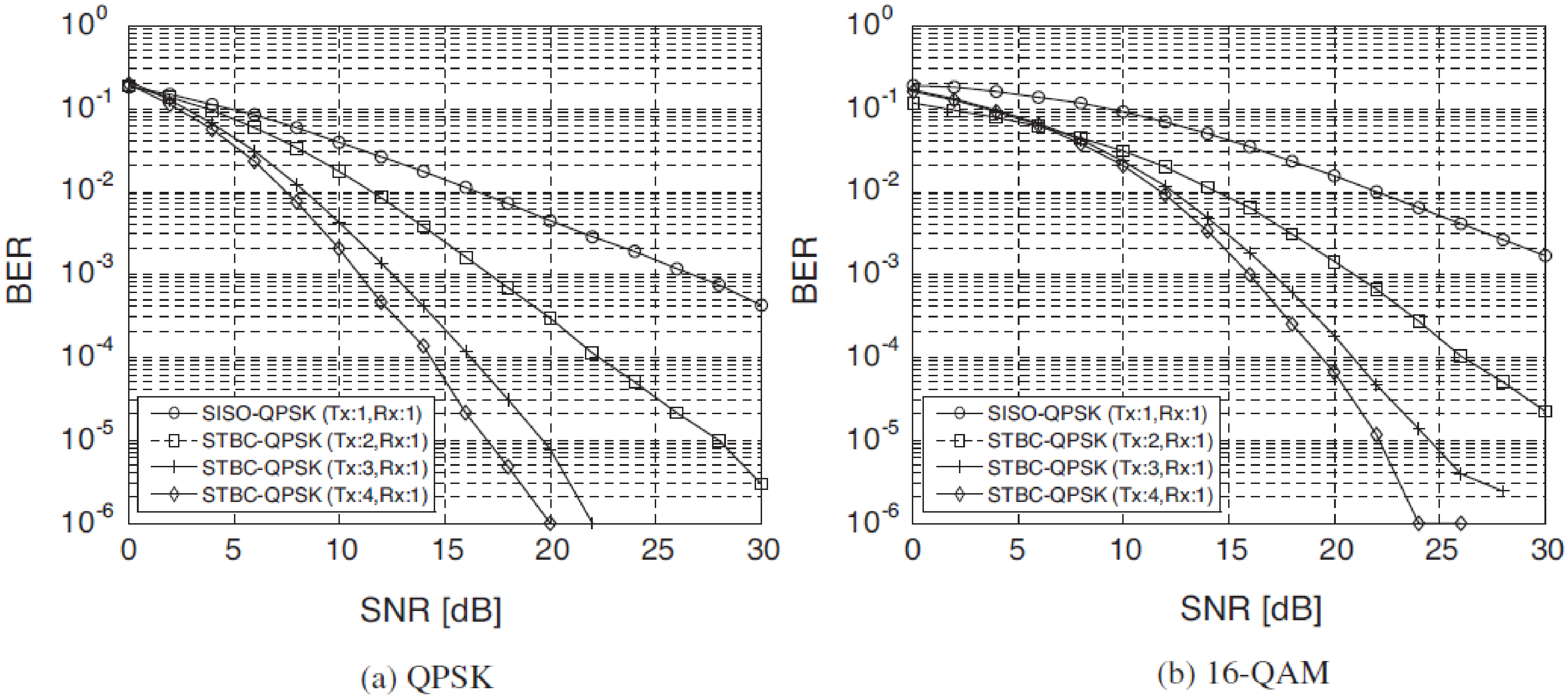
R=3/4

10.3.3 Uzay Zaman Kod Çözümü

$$[y_1\ y_2\ y_3\ y_4] = \sqrt{\frac{\mathrm{E_x}}{3\mathrm{N_0}}}[h_1\ h_2\ h_3]\begin{bmatrix} x_1 & -x_2^* & \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} \\ x_2 & x_1^* & \frac{x_3^*}{\sqrt{2}} & \frac{-x_3^*}{\sqrt{2}} \\ \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} & \frac{(-x_1-x_1^*+x_2-x_2^*)}{2} & \frac{(x_2+x_2^*+x_1-x_1^*)}{2} \end{bmatrix} + [z_1\ z_2\ z_3\ z_4] \quad (10.77)$$

$$\begin{bmatrix} \ \end{bmatrix} = \sqrt{\frac{E_x}{3N_0}} \begin{bmatrix} h_1 & h_3 & \frac{h_3}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \ \end{bmatrix}$$

- Tek alıcı, üç verici anten için ve karmaşık, yüksek oranlı bir STBC için
- (10.77), (10.78), (10.79), (10.80), (10.81),
- Daha fazla denklem manipülasyonu yapmak gerekiyor.
- Sonuçta yine denklemler oluşuyor
- Program 10.4 “STBC_3x4_simulation.m” to simulate 3x4 Space-Time Block Coding



(a) QPSK

(b) 16-QAM

Figure 10.9 BER performance of various space-time block codes.