MIMO-OFDM Wireless Communications with MATLAB®

SISO : $C = W \log_2 \left(1 + \frac{P|h|^2}{\sigma^2}\right)$

min(N_R, N_T) çarpan (MIMO)

Chapter 9. MIMO: Kanal Kapasitesi

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WILEY

Chapter 9. MIMO: Kanal Kapasitesi

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$\mathbb{H}_{N_R \times N_T}$ matris

rate factor $\min(N_R, N_T)$

Chapter 9. MIMO: Kanal Kapasitesi

9.1 Gerekli Matris Teorisi

$H_{N_R \times N_T}$

singular value decomposition (svd)

unitary: $UU^H = I$

$$\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H \tag{9.1}$$

$U_{N_R \times N_R}$

singular value

$\Sigma_{N_R \times N_T}$: diag$(\sigma_1, \sigma_2, \cdots \sigma_{N_{min}})$

$V = N_T \times N_T$

unitary: $VV^H = I$

$N_{min} = \min(N_R, N_T)$

$N_T \le N_R$

$N_{min} = N_T$

$$\begin{aligned}\mathbf{H} &= \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H \\ &= \underbrace{[\mathbf{U}_{N_{\min}}\ \mathbf{U}_{N_R-N_{\min}}]}_{\mathbf{U}} \underbrace{\begin{bmatrix} \boldsymbol{\Sigma}_{N_{\min}} \\ \mathbf{0}_{N_R-N_{\min}} \end{bmatrix}}_{\boldsymbol{\Sigma}} \mathbf{V}^H \\ &= \mathbf{U}_{N_{\min}} \boldsymbol{\Sigma}_{N_{\min}} \mathbf{V}^H \end{aligned} \tag{9.2}$$

$N_R \le N_T$

$N_{min} = N_R$

$$\begin{aligned}\mathbf{H} &= \mathbf{U} \underbrace{[\boldsymbol{\Sigma}_{N_{\min}}\ \mathbf{0}_{N_T-N_{\min}}]}_{\boldsymbol{\Sigma}} \underbrace{\begin{bmatrix} \mathbf{V}^H_{N_{\min}} \\ \mathbf{V}^H_{N_T-N_{\min}} \end{bmatrix}}_{\mathbf{V}^H} \\ &= \mathbf{U}\boldsymbol{\Sigma}_{N_{\min}} \mathbf{V}^H_{N_{\min}} \end{aligned} \tag{9.3}$$

9.1 Gerekli Matris Teorisi

$H = U \Sigma V^H$

$HH^H = U \Sigma V^H (U \Sigma V^H)^H = U \Sigma \underbrace{V^H V}_{I} \Sigma^H U^H = U \Sigma \Sigma^H U^H$

$$\mathbf{H}\mathbf{H}^H = \mathbf{U}\mathbf{\Sigma}\mathbf{\Sigma}^H\mathbf{U}^H = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^H \tag{9.4}$$

$\Sigma \Sigma^H = \Lambda \longrightarrow HH^H$ matrisinin özdeğerleri (diag)

eigen v.

$$\lambda_i = \begin{cases} \sigma_i^2, & \text{if } i = 1, 2, \cdots, N_{\min} \\ 0, & \text{if } i = N_{\min} + 1, \cdots, N_R. \end{cases} \tag{9.5}$$

σ_i gerçel sayı (her zaman)

$$\mathbf{H}\underbrace{[\mathbf{x}_1\ \mathbf{x}_2 \cdots \mathbf{x}_n]}_{\mathbf{X}} = \underbrace{[\mathbf{x}_1\ \mathbf{x}_2 \cdots \mathbf{x}_n]}_{\mathbf{X}}\mathbf{\Lambda}_{\text{non-}H} \tag{9.6}$$

$Q = U,$

$QQ^H = UU^H = I_{N_R}$

9.1 Gerekli Matris Teorisi

Frobenius norm :

$$\mathbf{H} = \mathbf{X}\mathbf{\Lambda}_{\text{non-}H}\mathbf{X}^{-1} \quad (9.7)$$

Total power gain of the channel.

$$\|\mathbf{H}\|_F^2 = \mathrm{Tr}(\mathbf{H}\mathbf{H}^H) = \sum_{i=1}^{N_R}\sum_{j=1}^{N_T}|h_{i,j}|^2. \quad (9.8)$$

trace

bütün değerlerin mutlak değer karelerinin toplamı.

$$\begin{aligned}\|\mathbf{H}\|_F^2 &= \|\mathbf{Q}^H\mathbf{H}\|_F^2 \\ &= \mathrm{Tr}\left(\mathbf{Q}^H\mathbf{H}\mathbf{H}^H\mathbf{Q}\right) \\ &= \mathrm{Tr}\left(\mathbf{Q}^H\mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^H\mathbf{Q}\right) \\ &= \mathrm{Tr}(\mathbf{\Lambda}) \\ &= \boxed{\sum_{i=1}^{N_{\min}}\lambda_i} \\ &= \boxed{\sum_{i=1}^{N_{\min}}\sigma_i^2}\end{aligned} \quad (9.9)$$

matris sırası trace sonucunu değiştirmez

I I

$\|H\|_F^2$

9.2 Deterministik MIMO Kanal kapasitesi

$\mathbf{x} = N_T \times 1$ vektör.

$$\mathbf{y} = \sqrt{\frac{E_x}{N_T}}\mathbf{H}\mathbf{x} + \mathbf{z} \qquad (9.10)$$

$N_R \times 1$ received signal vector.

$N_R \times 1$ noise.

$N_R \times N_T$

$z = (z_1, z_2, \dots, z_{N_R})^T$

$$\mathbf{R}_{xx} = E\{\mathbf{x}\mathbf{x}^H\}. \qquad (9.11)$$

Autocorrelation.
öz ilinti

$$\bar{R}_{xx} = \begin{bmatrix} E[x_1 x_1^*] & E[x_1 x_2^*] & \cdots & E[x_1 x_{N_T}^*] \\ \vdots & & & \\ E[x_{N_T} x_1^*] & & & \end{bmatrix}$$

$\in \mathbb{C}^{N_R \times 1}$

ZMCSCG

$\bar{z} \sim e^{j\theta} z$

$T_R(R_{xx}) = N_T$

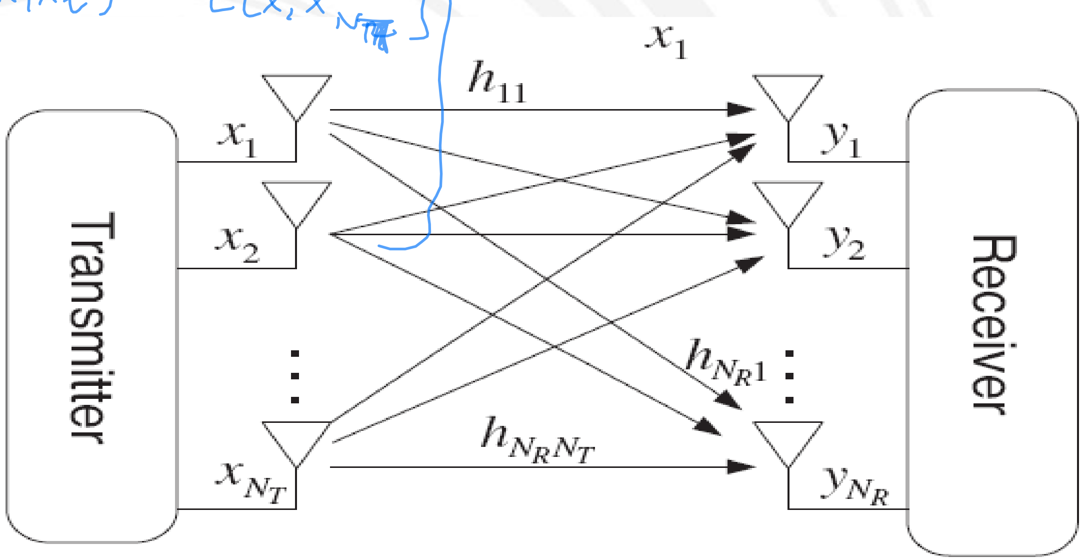


Figure 9.1 $N_R \times N_T$ MIMO system.

9.2.1 Vericide kanalın bilindiği durum

tx

rx

$$C = \max_{f(\mathbf{x})} I(\mathbf{x};\mathbf{y}) \text{ bits/channel use} \tag{9.12}$$

x'in pdf'i

mutual information.

differential entropy $H(y) = \int -f(y)\log_2(f(y))dy$

$$I(\mathbf{x};\mathbf{y}) = H(\mathbf{y}) - H(\mathbf{y}|\mathbf{x}) \tag{9.13}$$

conditional diff entropy.

$$H(\mathbf{y}|\mathbf{x}) = H(\mathbf{z}) \tag{9.14}$$

gürültü

$$I(\mathbf{x};\mathbf{y}) = H(\mathbf{y}) - \underline{H(\mathbf{z})} \tag{9.15}$$

9.2.1 Vericide kanalın bilindiği durum

$E[z] = 0$

$$
\begin{aligned}
\mathbf{R}_{yy} &= E\{\mathbf{y}\mathbf{y}^H\} = E\left\{\left(\sqrt{\frac{\mathrm{E_x}}{N_T}}\mathbf{H}\mathbf{x} + \mathbf{z}\right)\left(\sqrt{\frac{\mathrm{E_x}}{N_T}}\mathbf{x}^H\mathbf{H}^H + \mathbf{z}^H\right)\right\} \\
&= E\left\{\left(\frac{\mathrm{E_x}}{N_T}\mathbf{H}\mathbf{x}\mathbf{x}^H\mathbf{H}^H + \mathbf{z}\mathbf{z}^H\right)\right\} \\
&= \frac{\mathrm{E_x}}{N_T}E\{\mathbf{H}\mathbf{x}\mathbf{x}^H\mathbf{H}^H + \mathbf{z}\mathbf{z}^H\} \\
&= \frac{\mathrm{E_x}}{N_T}\mathbf{H}E\{\mathbf{x}\mathbf{x}^H\}\mathbf{H}^H + E\{\mathbf{z}\mathbf{z}^H\} \\
&= \frac{\mathrm{E_x}}{N_T}\mathbf{H}\mathbf{R}_{xx}\mathbf{H}^H + \mathrm{N_0}\mathbf{I}_{N_R}
\end{aligned}
\tag{9.16}
$$

$$
\begin{bmatrix}
\overset{N_0}{E[z_1 z_1^*]} & \overset{0}{E[z_1 z_2^*]} & \cdots & \overset{0}{E[z_1 z_{N_R}^*]} \\
\vdots & & & \\
\overset{0}{E[z_{N_R} z_1^*]} & \overset{0}{E[z_{N_R} z_2^*]} & \cdots & \underset{N_0}{E[z_{N_R} z_{N_R}^*]}
\end{bmatrix}
$$

$$
\begin{aligned}
H(\mathbf{y}) &= \log_2\{\det(\pi e \mathbf{R}_{yy})\} \\
H(\mathbf{z}) &= \log_2\{\det(\pi e \mathrm{N_0}\mathbf{I}_{N_R})\}
\end{aligned}
\tag{9.17}
$$

9.2.1 Vericide kanalın bilindiği durum

$\log_2(\det(\pi e R_{yy})) - \log(\det(\pi e N_0 I_{N_R}))$

$= \log_2(\det(\pi e R_{yy}) / \det(\pi e N_0 I_{N_R}))$

$$I(\mathbf{x};\mathbf{y}) = \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\mathrm{E_x}}{N_T \mathrm{N_0}} \mathbf{H}\mathbf{R}_{xx}\mathbf{H}^H\right) \text{ bps/Hz.} \tag{9.18}$$

MIMO kanal kapasitesi

$\log_2\left(1 + \frac{E_x}{N_0} |h|^2\right)$ SISO

$$C = \max_{Tr(\mathbf{R}_{xx}) = N_T} \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\mathrm{E_x}}{N_T \mathrm{N_0}} \mathbf{H}\mathbf{R}_{xx}\mathbf{H}^H\right) \text{ bps/Hz.} \tag{9.19}$$

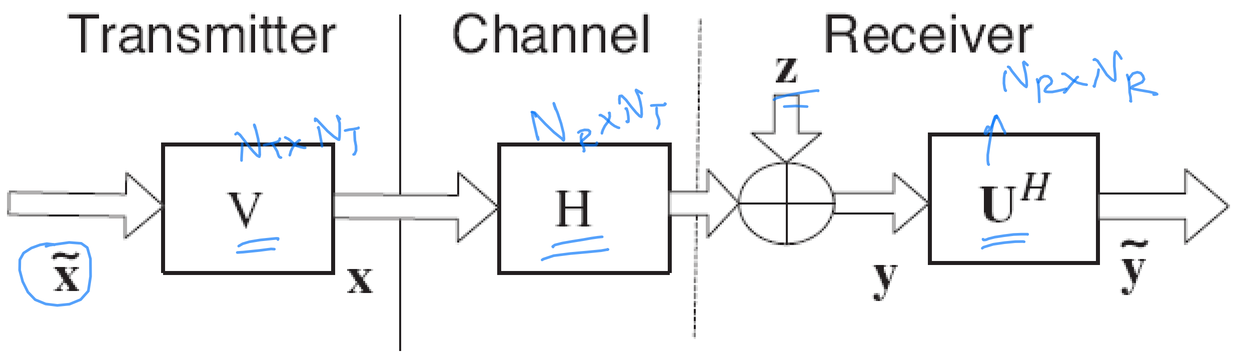


Figure 9.2 Modal decomposition when CSI is available at the transmitter side.

9.2.1 Vericide kanalın bilindiği durum (Precoding ve Post-processing)

(V) $\quad U^H$

$\tilde{y} = U^H H V \tilde{x} + U^H z$

diag

$$\tilde{\mathbf{y}} = \sqrt{\frac{\mathsf{E}_\mathrm{x}}{N_T}} \mathbf{U}^H \mathbf{H} \mathbf{V} \tilde{\mathbf{x}} + \tilde{\mathbf{z}} \tag{9.20}$$

$\underbrace{U^H U}_{I} \Sigma \underbrace{V^H V}_{I} \tilde{x} + U^H z = \Sigma \tilde{x} + \tilde{z} = \tilde{y}$

$$\boxed{\tilde{y}_i = \sqrt{\frac{\mathsf{E}_\mathrm{x}}{N_T}} \sqrt{\lambda_i} \tilde{x}_i + \tilde{z}_i,} \quad i = 1, 2, \cdots, r. \tag{9.21}$$

rank

MIMO kanal, r adet paralel SISO kanala çevrildi

$$\begin{bmatrix} \sqrt{\lambda_1}\,\tilde{x}_1 + \tilde{z}_1 \\ \sqrt{\lambda_2}\,\tilde{x}_2 + \tilde{z}_2 \\ \sqrt{\lambda_3}\,\tilde{x}_3 + \tilde{z}_3 \\ \vdots \\ \sqrt{\lambda_r}\,\tilde{x}_r + \tilde{z}_r \\ 0 \\ \vdots \\ 0 + \tilde{z}_{N_R} \end{bmatrix}_{N_R \times 1}$$

$$\boxed{C_i(\gamma_i) = \log_2\left(1 + \frac{\mathsf{E}_\mathrm{x}\gamma_i}{N_T \mathsf{N}_0}\lambda_i\right),} \quad i = 1, 2, \cdots, r. \tag{9.22}$$

$\gamma_i = E\left[|x_i|^2\right]$

$$E\{\mathbf{x}^H\mathbf{x}\} = \sum_{i=1}^{N_T} E\left\{|x_i|^2\right\} = N_T. \tag{9.23}$$

toplam güç

9.2.1 Vericide kanalın bilindiği durum (Uzamsal sanal paralel kanallar)

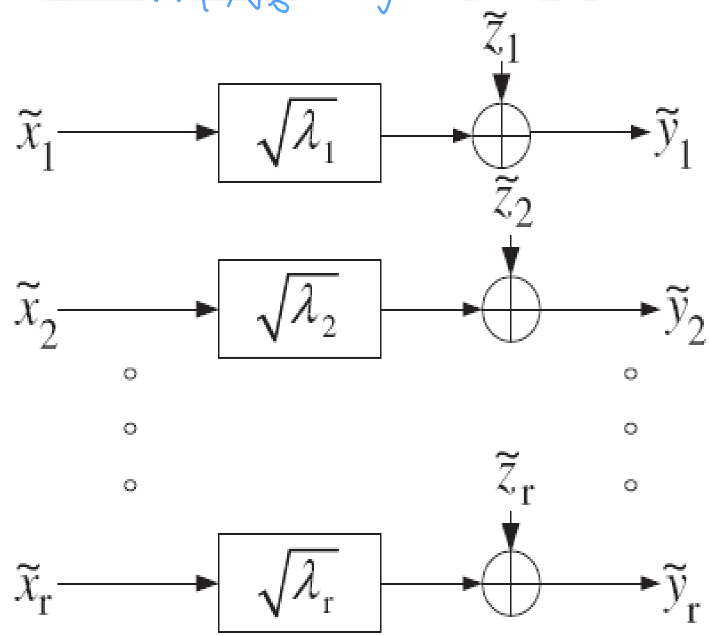
$\sigma_i^2 = \lambda_i$

$$C = \max_{\{\gamma_i\}} \sum_{i=1}^{r} \log_2\left(1+\frac{E_x \gamma_i}{N_T N_0}\lambda_i\right)$$

subject to

$$\sum_{i=1}^{r} \gamma_i = N_T$$

λ_i'si daha yüksek olan kanallara daha çok güç ayrılabilir.



Parallel SISO channels
spatial channels
uzamsal kanallar,

Figure 9.3 The r virtual SISO channels obtained from the modal decomposition of a MIMO channel.

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9.2.1 Vericide kanalın bilindiği durum (Su doldurma)

türev

$$C = \sum_{i=1}^{r} C_i(\gamma_i) = \sum_{i=1}^{r} \log_2\left(1 + \frac{\mathsf{E_x}\gamma_i}{N_T \mathsf{N_0}}\lambda_i\right) \tag{9.24}$$

→ max

$$C = \max_{\{\gamma_i\}} \sum_{i=1}^{r} \log_2\left(1 + \frac{\mathsf{E_x}\gamma_i}{N_T \mathsf{N_0}}\lambda_i\right) \tag{9.25}$$

lagrange multiplier.

$$\gamma_i^{opt} = \left(\mu - \frac{N_T \mathsf{N_0}}{\mathsf{E_x}\lambda_i}\right)^+, \quad i = 1, \cdots, r \tag{9.26}$$

→ Water filing.

taban seviyesi

$$\sum_{i=1}^{r} \gamma_i^{opt} = N_T. \tag{9.27}$$

$\boxed{\mu : \text{binary search}}$

$$(x)^+ = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}. \tag{9.28}$$

9.2.1 Vericide kanalın bilindiği durum (Su doldurma görseli)

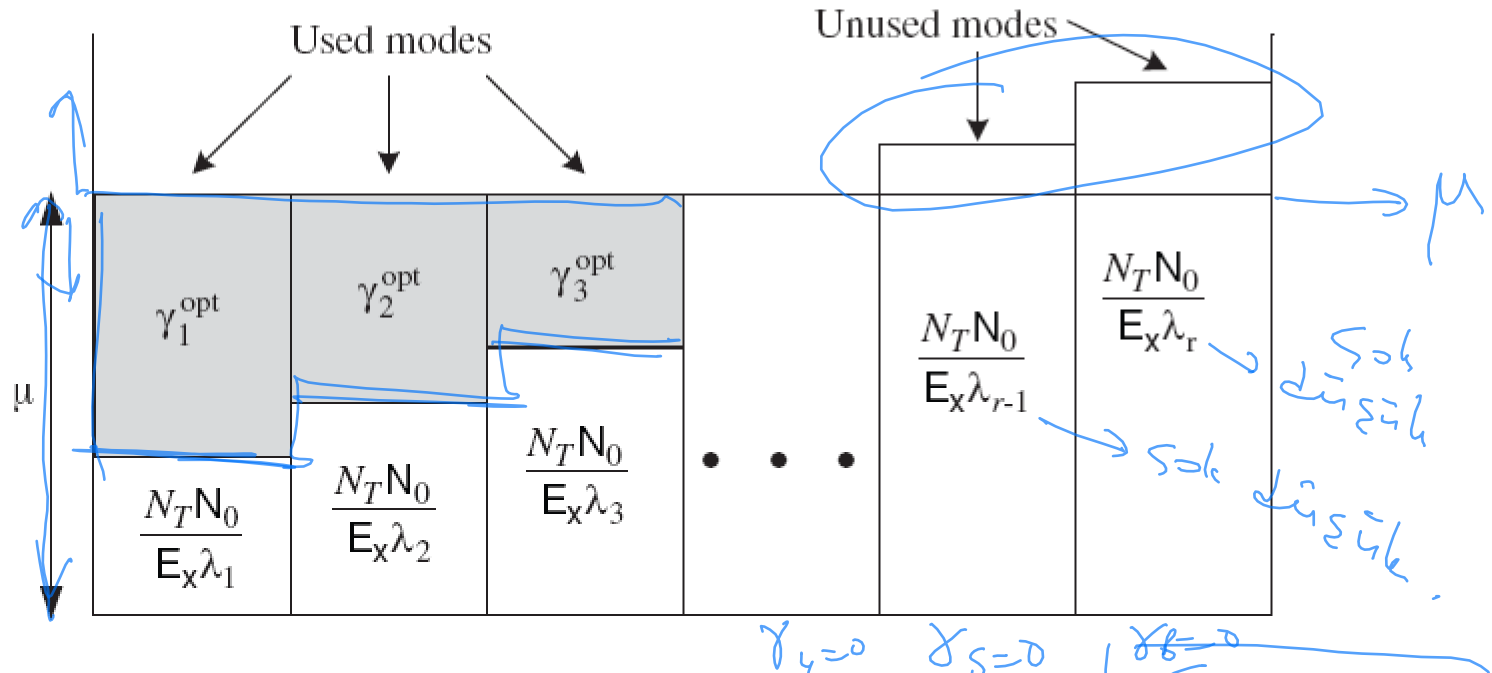


Figure 9.4 Water-pouring power allocation algorithm.

$\boxed{\sum \gamma_i = N_T}$

9.2.2 Vericide kanalın bilinmediği durum

power spread equally to all transmit antenn.

$$\mathbf{R}_{xx} = \mathbf{I}_{N_T} \tag{9.29}$$

$R_{xx} = I$

$$C = \log_2 \det\left(\mathbf{I}_{N_R} + \frac{E_x}{N_T N_0}\mathbf{H}\mathbf{H}^H\right). \tag{9.30}$$

$\rightarrow Q \Lambda Q^H$ eigen decomposition.

$$C = \log_2 \det\left(\mathbf{I}_{N_R} + \frac{E_x}{N_T N_0}\mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^H\right) = \log_2 \det\left(\mathbf{I}_{N_R} + \frac{E_x}{N_T N_0}\mathbf{\Lambda}\right)$$

diag diag

$Q Q^H \Lambda$

I

$\det(I_m + AB) = \det(I_n + BA)$

$$C = \sum_{i=1}^{r} \log_2\left(1 + \frac{E_x}{N_T N_0}\lambda_i\right) \tag{9.31}$$

rank of H

$(\gamma_i = 1)$

$Q \Lambda Q^H \rightarrow Q Q^H \Lambda$

I

Paralel siso kanallar.

9.2.2 Vericide kanalın bilinmediği durum

$\|\mathbf{H}\|_F^2 = \sum_{i=1}^{r} \lambda_i = \zeta$ sabit ise, ve H full rank ise ($N_T = N_R = N$) ve $r = N$

$$\lambda_i = \frac{\zeta}{N}, \quad i = 1, 2, \cdots, N. \tag{9.32}$$

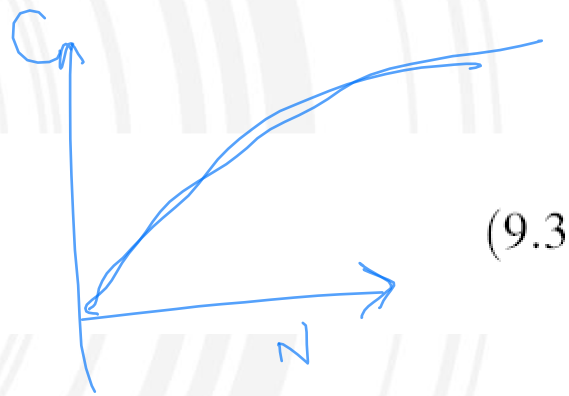
→ böyle olduğunda (kapasite maks olur)

$$\mathbf{H}\mathbf{H}^H = \mathbf{H}^H\mathbf{H} = \frac{\zeta}{N}\mathbf{I}_N \tag{9.33}$$

→ kanal ortogonal.

$$C = N \log_2\left(1 + \frac{\zeta \mathrm{E_x}}{\mathrm{N_0} N}\right). \tag{9.34}$$

$N \times N$



9.2.3 SIMO ve MISO Kanal Kapasitesi (MIMO'nun özel durumu)

$C = \log_2 \det\left(I_{N_R} + \frac{E_x}{N_T N_0} H \cancel{R_{xx}} H^H\right)$

$N_T = 1$, $h \in N_{R\times 1}$ $\quad r = 1 \longrightarrow \lambda_1 = \|h\|_F^2$

$$C_{SIMO} = \log_2\left(1 + \frac{\mathsf{E_x}}{\mathsf{N_0}}\|\mathbf{h}\|_F^2\right). \tag{9.35}$$

SIMO

$|h_i|^2 = 1$ olsa $\forall i = 1, \ldots, N_R$

$$C_{SIMO} = \log_2\left(1 + \frac{\mathsf{E_x}}{\mathsf{N_0}} N_R\right). \tag{9.36}$$

alıcıda SNR artar (QAM derecesi artırılabilir)

MISO

CSIT yoksa

$N_R = 1$, $h_{1\times N_T}$, $r = 1$, $\lambda_1 = \|h\|_F^2$

$$C_{MISO} = \log_2\left(1 + \frac{\mathsf{E_x}}{N_T \mathsf{N_0}}\|\mathbf{h}\|_F^2\right). \tag{9.37}$$

CSIT yoksa verici kanalı bilmiyorsa.

$|h_i|^2 = 1$ olsa

$$C_{MISO} = \log_2\left(1 + \frac{\mathsf{E_x}}{\mathsf{N_0}}\right). \tag{9.38}$$

$\frac{\cancel{N_T}}{\cancel{N_T}}$

SISO gibi. N_T işe yaramıyor gibi ama pratikte space time coding diversity. BER↓

9.2.3 SIMO ve MISO Kanal Kapasitesi (MIMO'nun özel durumu)

MISO: CSIT Varsa h is known.

$\frac{\mathbf{h}^H}{\|\mathbf{h}\|} x$ is transmitted

$$y = \sqrt{\mathrm{E_x}}\,\mathbf{h} \cdot \frac{\mathbf{h}^H}{\|\mathbf{h}\|} x + z = \sqrt{\mathrm{E_x}}\,\|\mathbf{h}\| x + z \qquad (9.39)$$

kanal

t_x

$h h^H = \|h\|^2$

$\sqrt{|h_{11}|^2 + |h_{21}|^2 + \cdots |h_{N_T 1}|^2}$

$$C_{MISO} = \log_2\left(1 + \frac{\mathrm{E_x}}{\mathrm{N_0}}\|\mathbf{h}\|_F^2\right) = \log_2\left(1 + \frac{\mathrm{E_x}}{\mathrm{N_0}} N_T\right). \qquad (9.40)$$

$|h_{i1}|^2 = 1$ olsa.

logaritmik kazanç.

Random

9.3 Rastgele MIMO Kanal Kapasitesi (Beklenen değer ve Kesinti Olasılığı)

H: ergodik

ortalama

$$\overline{C} = E\{C(\mathbf{H})\} = E\left\{ \max_{\mathrm{Tr}(\mathbf{R}_{xx})=N_T} \log_2 \det\left(\mathbf{I}_{N_R} + \frac{E_x}{N_T N_0} \mathbf{H}\mathbf{R}_{xx}\mathbf{H}^H \right) \right\} \qquad (9.41)$$

$$\overline{C_{OL}} = E\left\{ \underbrace{\sum_{i=1}^{r} \log_2\left(1 + \frac{E_x}{N_T N_0}\lambda_i\right)}_{} \right\}. \qquad (9.42)$$

open loop

no CSIT → $R_{xx} = I_{N_T}$

$$\overline{C}_{CL} = E\left\{ \max_{\sum_{i=1}^{r}\gamma_i = N_T} \sum_{i=1}^{r} \log_2\left(1 + \frac{E_x}{N_T N_0}\gamma_i\lambda_i\right) \right\} \qquad (9.43)$$

closed loop

CSIT var.

waterfilling

$$= E\left\{ \sum_{i=1}^{r} \log_2\left(1 + \frac{E_x}{N_T N_0}\gamma_i^{opt}\lambda_i\right) \right\}. \qquad (9.44)$$

outage → kesinti

$$P_{out}(R) = \Pr(C(\mathbf{H}) < R) \qquad (9.45)$$

$= 0.1$

anlık kapasitenin bir R değerinden küçük olma olasılığı.

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Kodlar

- Program 9.1 “Ergodic_Capacity_CDF.m” for ergodic capacity of MIMO channel
- Program 9.2 “Ergodic_Capacity_vs_SNR.m” for ergodic channel capacity vs. SNR in Figure 9.6.
- Program 9.3 “OL_CL_Comparison.m” for Ergodic channel capacity: open-loop vs. closedloop
- Program 9.4 “Water_Pouring” for water-pouring algorithm
- Program 9.5 “Ergodic_Capacity_Correlation.m:” Channel capacity reduction due to correlation

9.3 Rastgele MIMO Kanal Kapasitesi: CDF

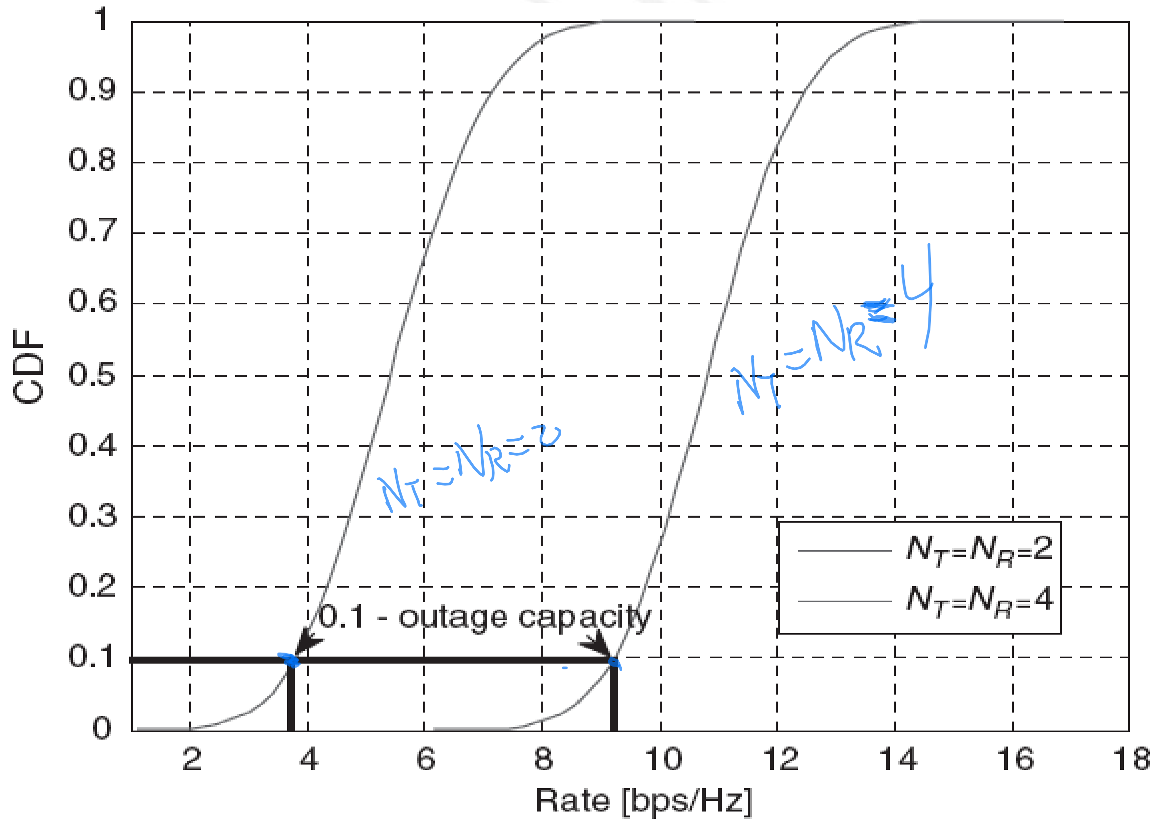

Figure 9.5 Distribution of MIMO channel capacity (SNR = 10dB; CSI is not available at the transmitter side).

9.3 Rastgele MIMO Kanal Kapasitesi: vs SNR

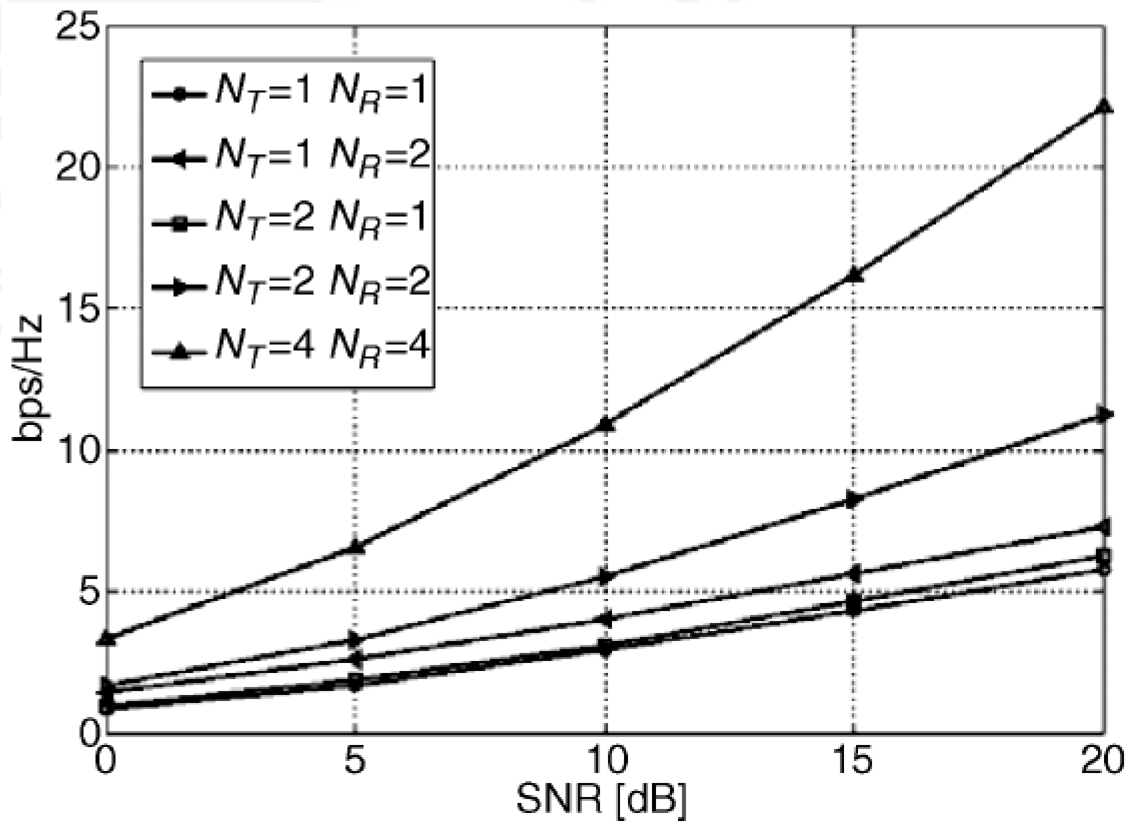

Figure 9.6 Ergodic MIMO channel capacity when CSI is not available at the transmitter.

9.3 Rastgele MIMO Kanal Kapasitesi

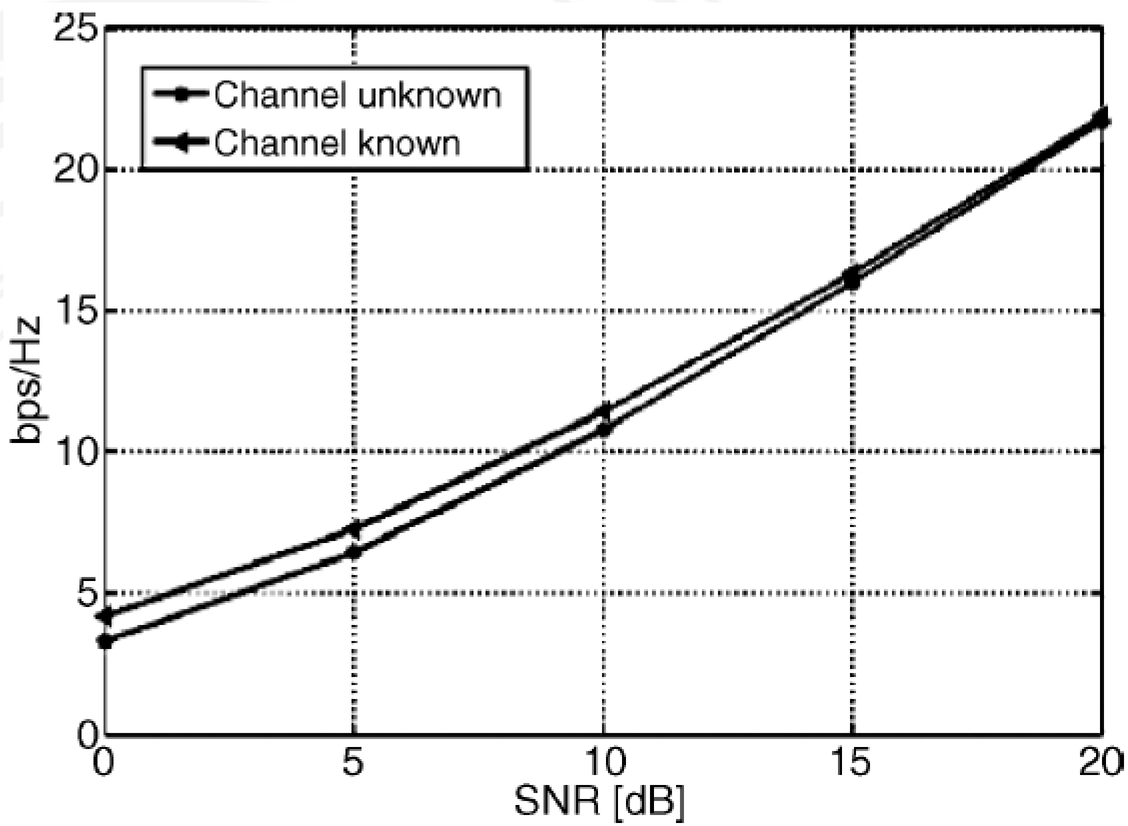

Figure 9.7 Ergodic channel capacity: $N_T = N_R = 4$.

9.3 Rastgele MIMO Kanal Kapasitesi: Antenler arası korelasyon var ise

SNR yüksek olduğunda

$$\underbrace{C \approx \max_{\mathrm{Tr}(\mathbf{R}_{xx})=N} \log_2 \det(\mathbf{R}_{xx})}_{\leq 0} + \underbrace{\log_2 \det\left(\frac{\mathsf{E}_x}{N\mathsf{N}_0}\mathbf{H}_w\mathbf{H}_w^H\right)}_{\text{uncorrelated MIMO channel capacity}} \tag{9.46}$$

max kapasite $R_{xx} = I$ olduğunda (uncorrelated)

$$\boxed{\mathbf{H} = \underset{rx}{\mathbf{R}_r^{1/2}}\,\underset{uncor.}{\mathbf{H}_w}\,\underset{tx}{\mathbf{R}_t^{1/2}}} \tag{9.47}$$

$$C = \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\mathsf{E}_x}{N_T\mathsf{N}_0}\underbrace{\mathbf{R}_r^{1/2}\mathbf{H}_w\mathbf{R}_t\mathbf{H}_w^H\mathbf{R}_r^{H/2}}\right). \tag{9.48}$$

HH^H

$R_r^{1/2} H_w R_t^{1/2}$ $\quad R_t^{1/2\,H} H_w^H R_r^{1/2\,H}$

$$C \approx \underbrace{\log_2 \det\left(\frac{\mathsf{E}_x}{N_T\mathsf{N}_0}\mathbf{H}_w\mathbf{H}_w^H\right)}_{\text{uncorrelated}} + \underbrace{\log_2 \det(\mathbf{R}_r)}_{\leq 0} + \underbrace{\log_2 \det(\mathbf{R}_t)}_{\leq 0}. \tag{9.49}$$

$R_r H_w H_w^H R_t$

9.3 Rastgele MIMO Kanal Kapasitesi
Antenler arası korelasyon var ise

$$\log_2 \det(\mathbf{R}_r) + \log_2 \det(\mathbf{R}_t). \tag{9.50}$$

$$\det(\mathbf{R}) = \prod_{i=1}^{N} \lambda_i. \tag{9.51}$$

$$\left(\Pi_{i=1}^{N} \lambda_i\right)^{\frac{1}{N}} \le \frac{1}{N}\sum_{i=1}^{N} \lambda_i = 1. \tag{9.52}$$

$$\boxed{\sum \lambda_i = N_T}$$

geometrik ≤ aritmetik

$$\log_2 \det(\mathbf{R}) \le 0 \tag{9.53}$$

$\det(R) \le 1$

$$\boxed{\log \det(R) \le 0}$$

9.3 Rastgele MIMO Kanal Kapasitesi Antenler arası korelasyon var ise

$$\mathbf{R}_t = \begin{bmatrix} 1 & 0.76e^{j0.17\pi} & 0.43e^{j0.35\pi} & 0.25e^{j0.53\pi} \\ 0.76e^{-j0.17\pi} & 1 & 0.76e^{j0.17\pi} & 0.43e^{j0.35\pi} \\ 0.43e^{-j0.35\pi} & 0.76e^{-j0.17\pi} & 1 & 0.76e^{j0.17\pi} \\ 0.25e^{-j0.53\pi} & 0.43e^{-j0.35\pi} & 0.76e^{-j0.17\pi} & 1 \end{bmatrix} \quad (9.54)$$

9.3 Rastgele MIMO Kanal Kapasitesi
Antenler arası korelasyon var ise

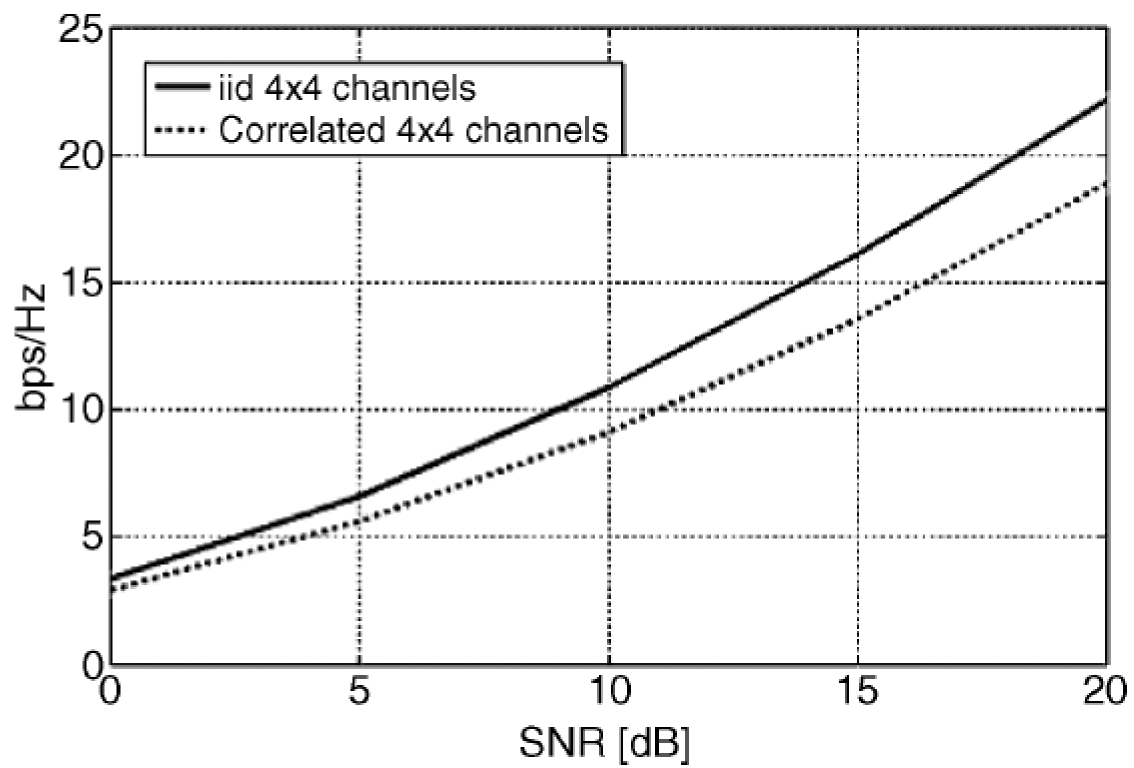

Figure 9.8 Capacity reduction due to the channel correlation.