# MIMO-OFDM Wireless Communications with MATLAB®

Chapter 7. PAPR Reduction

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Peak-to-Average Power Reduction

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# Chapter 7. PAPR Reduction

- 7.1 INTRODUCTION TO PAPR
  - 7.1.1 Definition of PAPR
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  - 7.2.4 Partial Transmit Sequence
  - 7.2.5 Tone Reservation
  - 7.2.6 Tone Injection
  - 7.2.7 DFT Spreading

# Chapter 7. PAPR Azaltımı
## 7.1 PAPR'a Giriş

$$IBO = 10\log_{10}\frac{P_{in}^{max}}{P_{in}}, \quad OBO = 10\log_{10}\frac{P_{out}^{max}}{P_{out}} \tag{7.1}$$

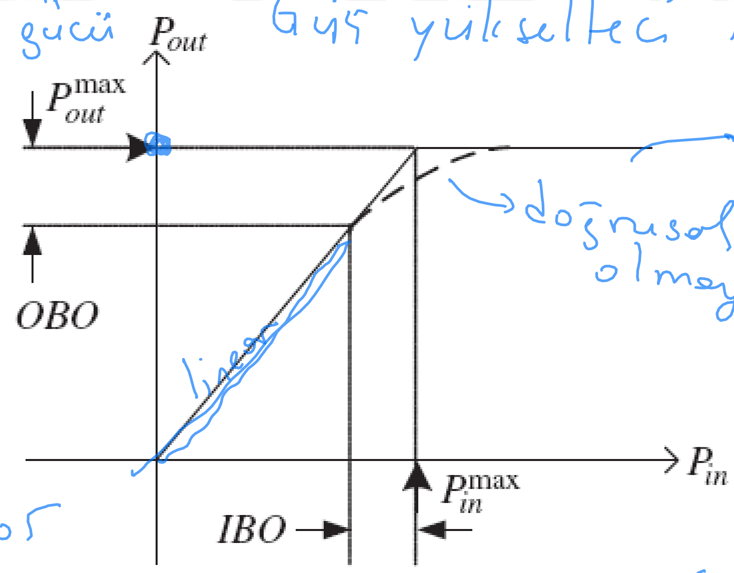


**Figure 7.1** Input-output characteristic of an HPA.



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# 7.1.1 PAPR Tanımı

$$\tilde{s}(t) = \sum_{k} a[n] g(t - kT_s) \qquad (7.2)$$

$$s(t) = \sqrt{2}\,\mathrm{Re}\{(\tilde{s}_I(t) + j\tilde{s}_Q(t))e^{j2\pi f_c t}\} \qquad (7.3)$$

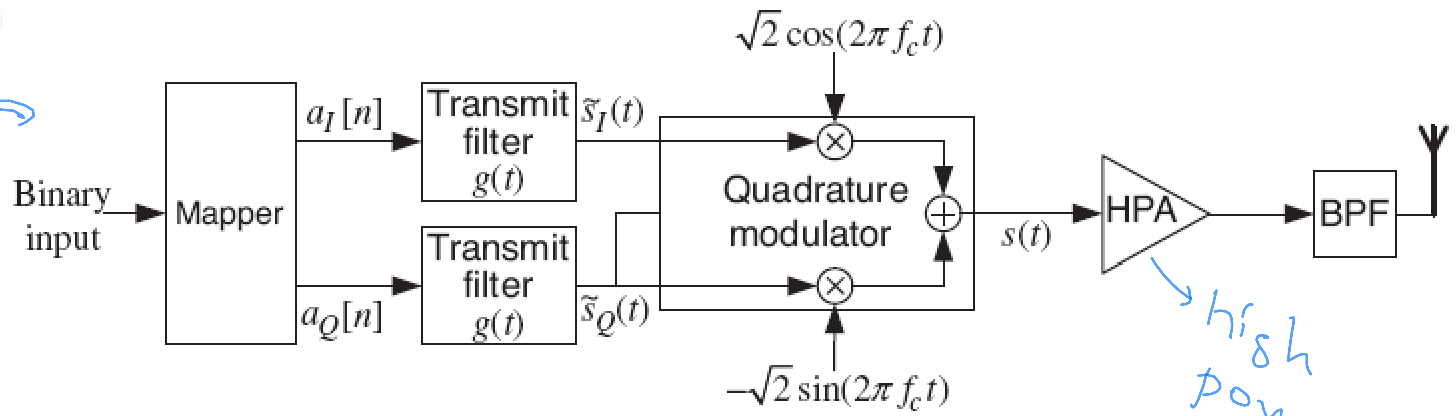


**Figure 7.2** Passband PAM transmitter.

# 7.1.1.1 Tepe-Ortalama Zarf Gücü Oranı (PMEPR)

peak to mean envelope power ratio
baseband

$$PMEPR\{\tilde{s}(t)\} = \frac{\max|\tilde{s}(t)|^2}{E\left\{|\tilde{s}(t)|^2\right\}} \quad (7.4)$$

# 7.1.1.2 Tepe Zarf Gücü (PEP)

peak envelope power

$$PEP\{\tilde{s}(t)\} = \max|\tilde{s}(t)|^2 \quad (7.5)$$

$E\{|\tilde{s}(t)|^2\} = 1$ ise PMEPR = PEP
normalize

# 7.1.1.3 Tepe-Ortalama Güç Oranı (PAPR)

$$PAPR\{\tilde{s}(t)\} = \frac{\max\left|\mathrm{Re}\left(\tilde{s}(t)e^{j2\pi f_c t}\right)\right|^2}{E\left\{\left|\mathrm{Re}\left(\tilde{s}(t)e^{j2\pi f_c t}\right)\right|^2\right\}} = \frac{\max|s(t)|^2}{E\left\{|s(t)|^2\right\}} \tag{7.6}$$

$$\text{Passband condition :} \quad CF = \sqrt{PAPR} \tag{7.7a}$$

$$\text{Baseband condition :} \quad CF = \sqrt{PMEPR} \tag{7.7b}$$

$$f_{Z_n}(z) = \frac{z}{\sigma^2} e^{-\frac{z^2}{2\sigma^2}} = 2ze^{-z^2}, \quad n = 0, 1, 2, \cdots, N-1 \tag{7.8}$$



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# 7.1.1.3 Tepe-Ortalama Güç Oranı (PAPR)

$$\begin{aligned} F_{\mathbf{Z}_{\max}}(z) &= P(\mathbf{Z}_{\max} < z) \\ &= P(\mathbf{Z}_0 < z) \cdot P(\mathbf{Z}_1 < z) \ldots P(\mathbf{Z}_{N-1} < z) \\ &= \left(1 - e^{-z^2}\right)^N \end{aligned} \tag{7.9}$$

$$\begin{aligned} \tilde{F}_{\mathbf{Z}_{\max}}(z) &= P(\mathbf{Z}_{\max} > z) \\ &= 1 - P(\mathbf{Z}_{\max} \leq z) \\ &= 1 - F_{\mathbf{Z}_{\max}}(z) \\ &= 1 - \left(1 - e^{-z^2}\right)^N \end{aligned} \tag{7.10}$$

$$F_Z(z) \approx \left(1 - e^{-z^2}\right)^{\alpha N} \tag{7.11}$$

# 7.1.1.3 Tepe-Ortalama Güç Oranı (PAPR)

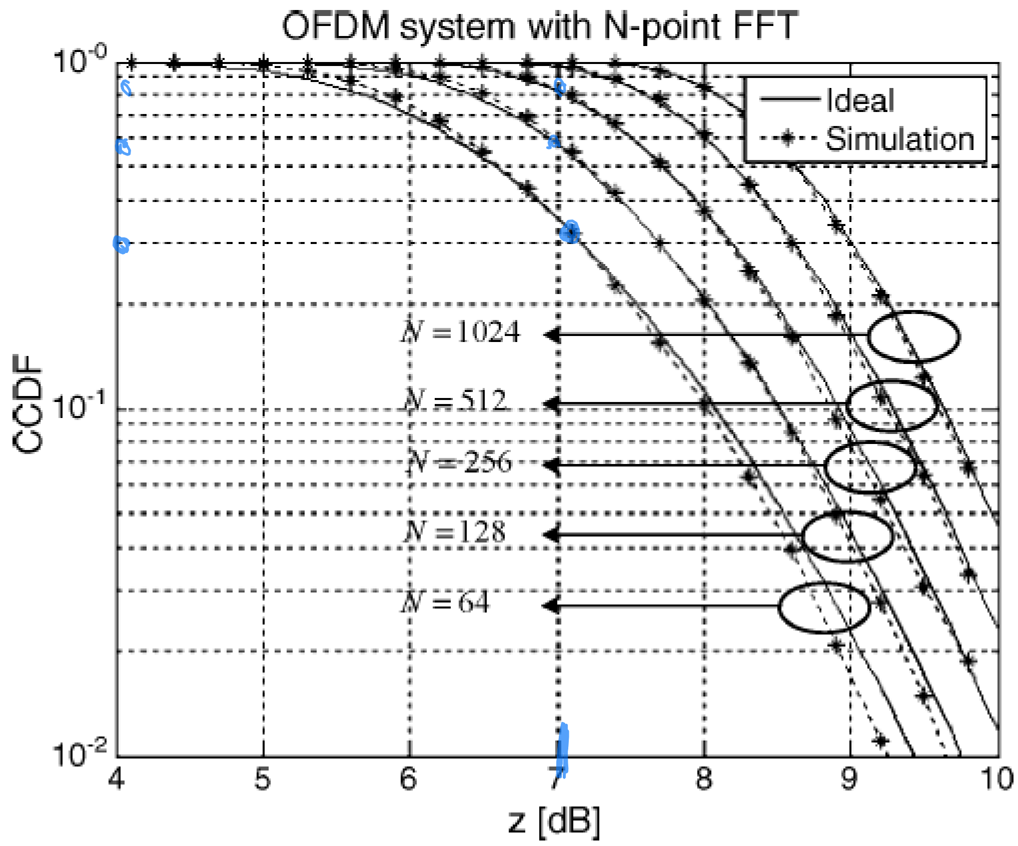


**Figure 7.3** CCDFs of OFDM signals with $N = 64, 128, 256, 512$, and 1024.



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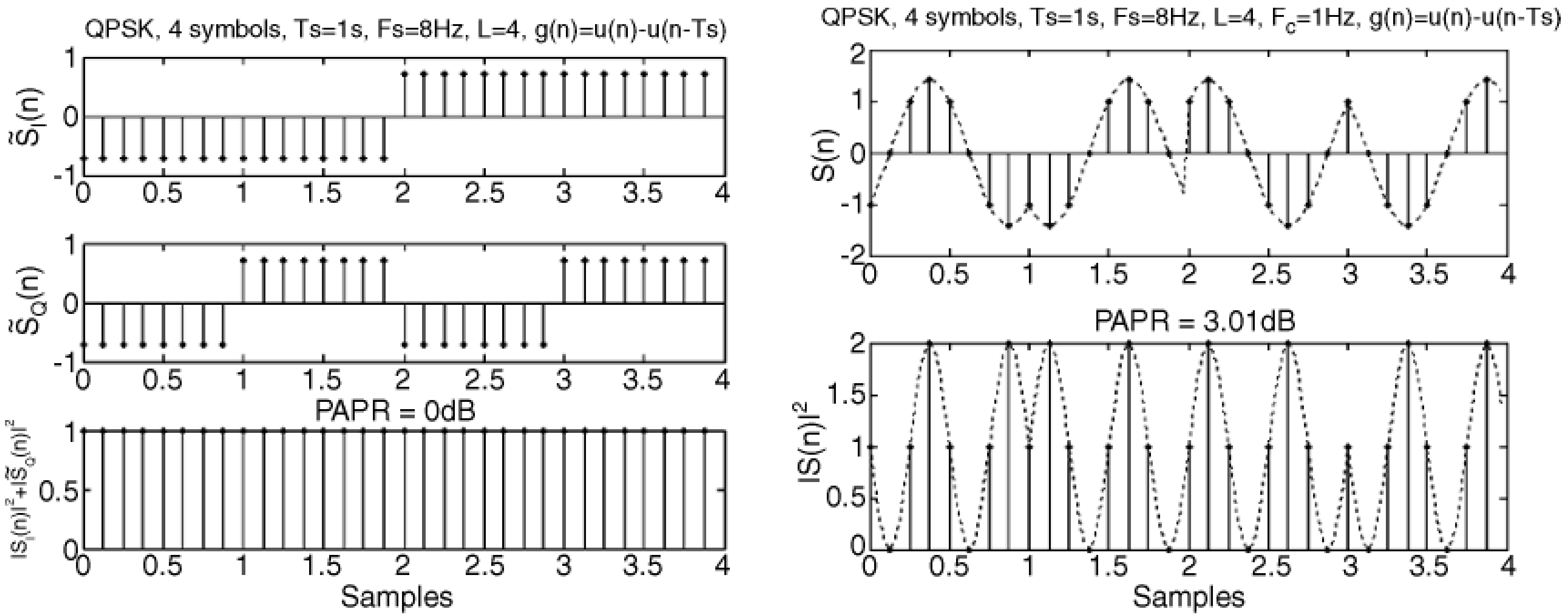
# 7.1.1.3 Tepe-Ortalama Güç Oranı (PAPR)




(a) Baseband signal (b) Passband signal

**Figure 7.4** Baseband/passband signals for QPSK-modulated symbols.

# 7.1.2 Distribution of OFDM Signal

ifft

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j\frac{2\pi}{N}kn} \quad (7.12)$$

Re[x[n]] ~ N

Im[x[n]] ~ N

QAM sembollen: |x[n]| ~ Rayleigh

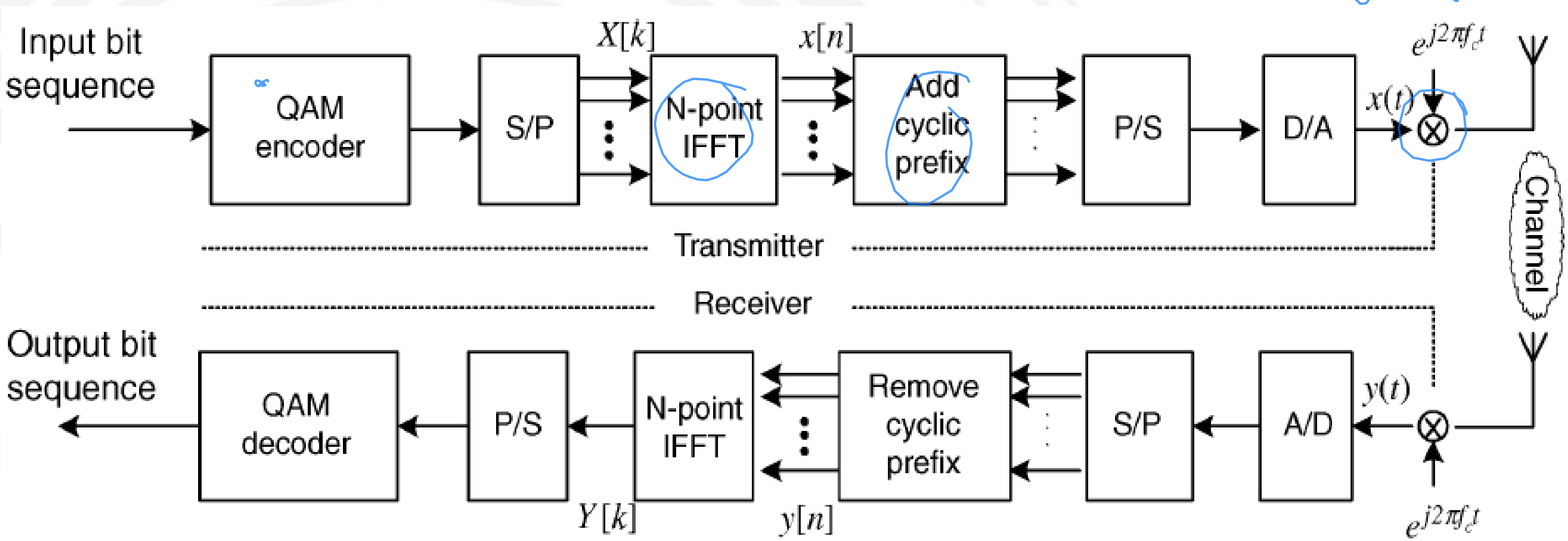



**Figure 7.5** Block diagram of OFDM system.

# 7.1.2 OFDM Sinyal Büyüklüğünün Dağılımı

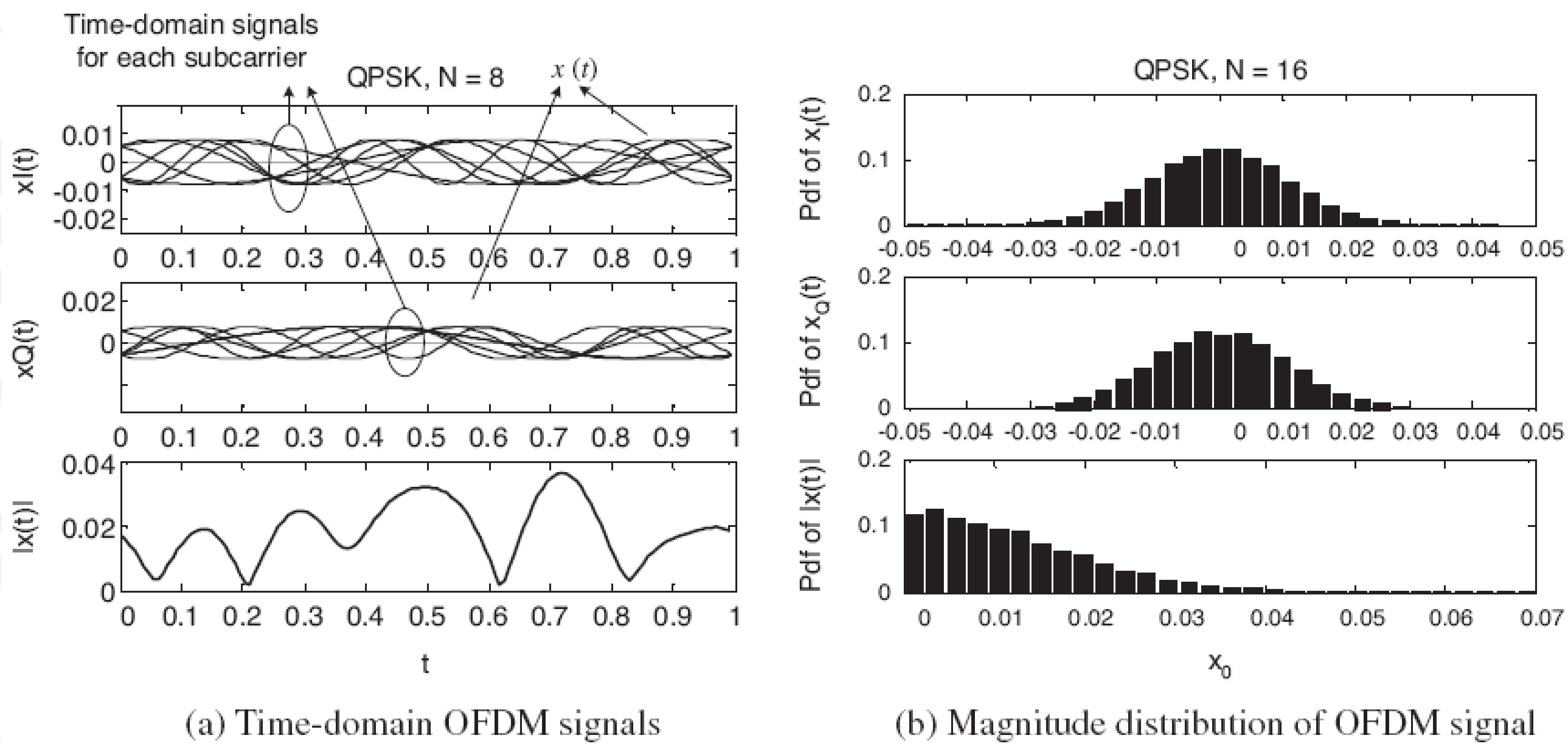



**Figure 7.6** Characteristics of time-domain QPSK/OFDM signals: $N = 8$ and 16.

# 7.1.4 Kırpma ve SQNR

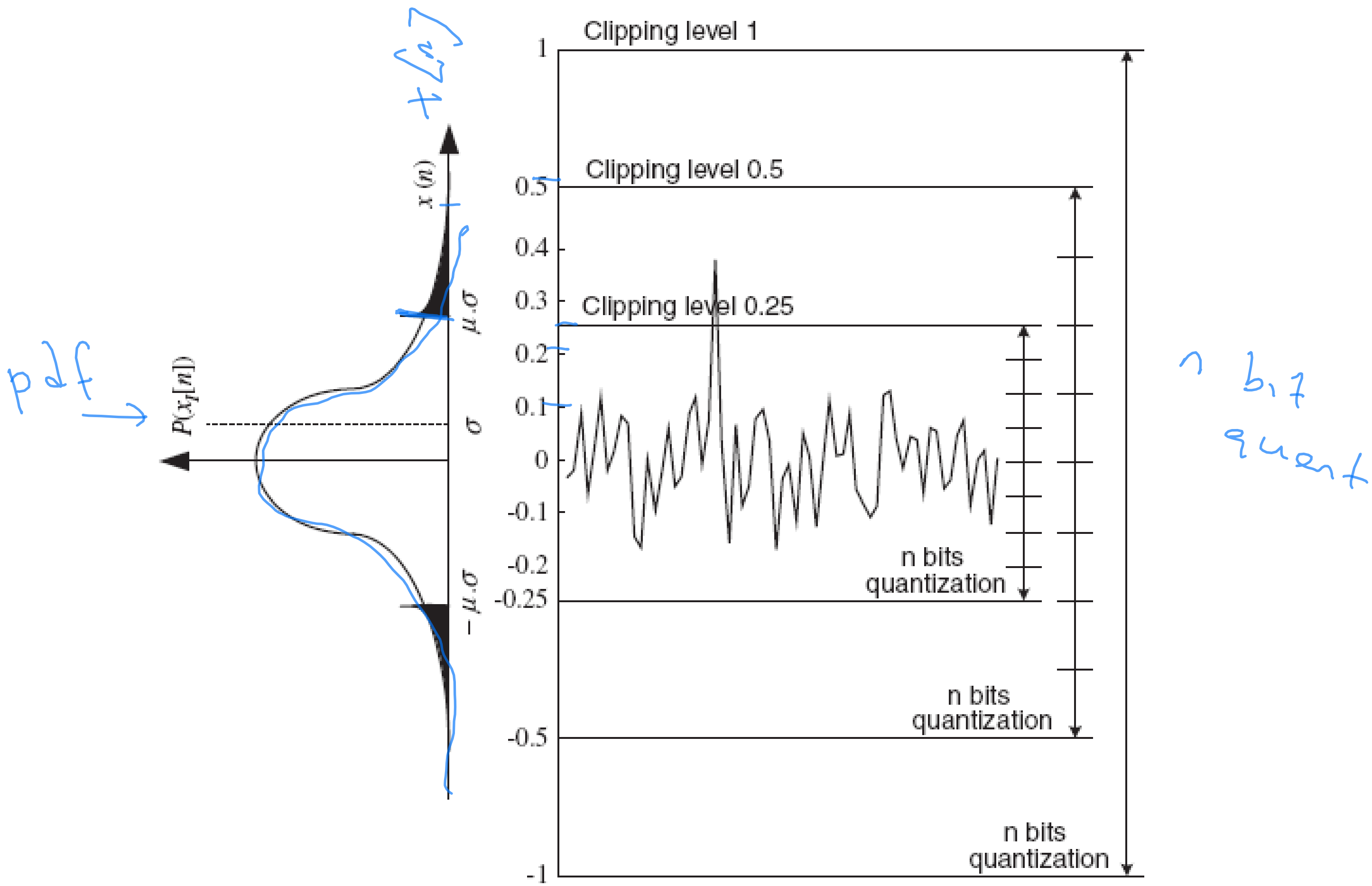



**Figure 7.11** Probabilistic distribution of the real part of a time-domain OFDM signal.



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# 7.1.4 Kırpma ve SQNR

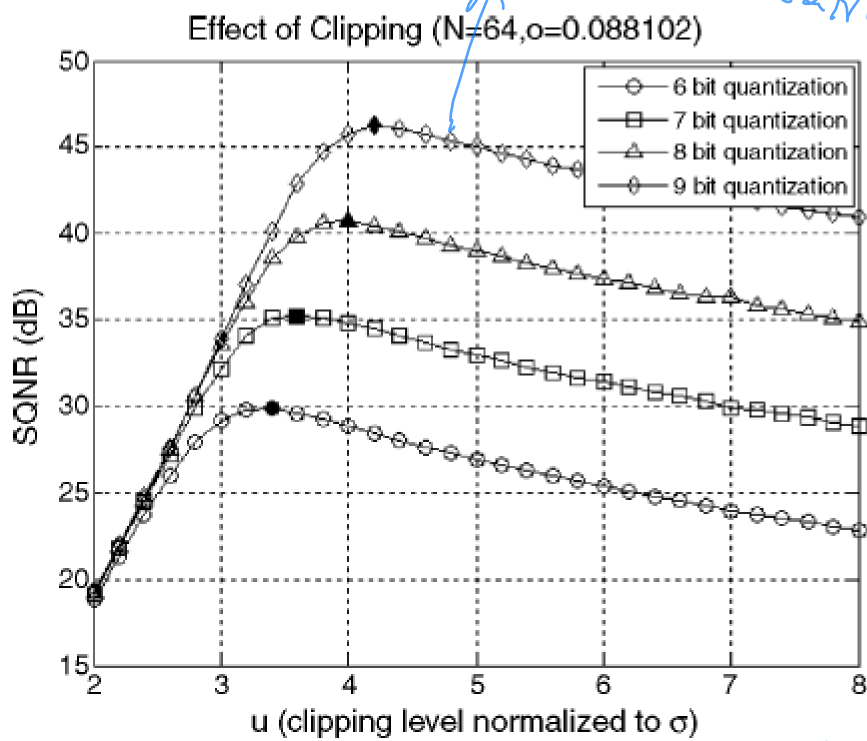



**Figure 7.12** SQNR of quantized OFDM signal against the clipping level.

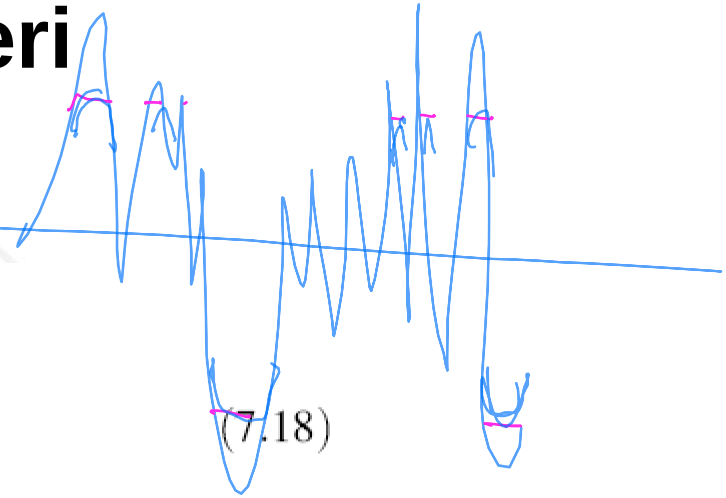
# Kodlar

- Program 7.1 (“plot_CCDF.m”).
- Program 7.2 (“PAPR”) for PAPR calculation
- Program 7.3 (“mapper”) for PSK/QAM
- Program 7.4 “single_carrier_PAPR.m”
- Program 7.5 “modulation()”
- Program 7.6 “OFDM_signal.m”

7-7
7-8
7-9
7-10

# 7.2 PAPR Azaltma Teknikleri

## 7.2.1 Kırpma ve Filtreleme



$$x_c^p[m] = \begin{cases} -A & x^p[m] \le -A \\ x^p[m] & |x^p[m]| < A \\ A & x^p[m] \ge A \end{cases} \quad (7.18)$$

$$x_c^p[m] = \begin{cases} x^p[m] & \text{if } |x^p[m]| < A \\ \dfrac{x^p[m]}{|x^p[m]|} \cdot A & \text{otherwise} \end{cases} \quad (7.19)$$

# 7.2.1 Kırpma ve Filtreleme

L oversampling factor.
N ifft

$$CR = \frac{A}{\sigma}$$

A → clipping level
σ → rms değeri (std)

passband

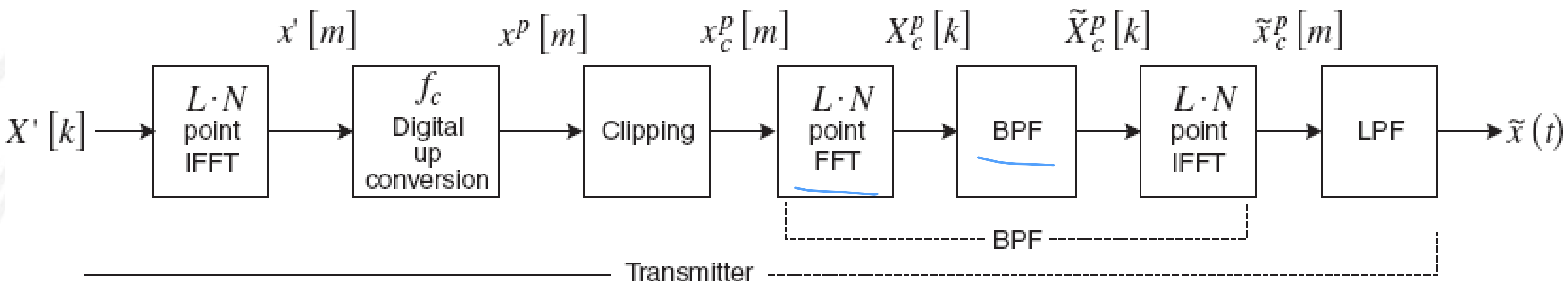



**Figure 7.13** Block diagram of a PAPR reduction scheme using clipping and filtering.

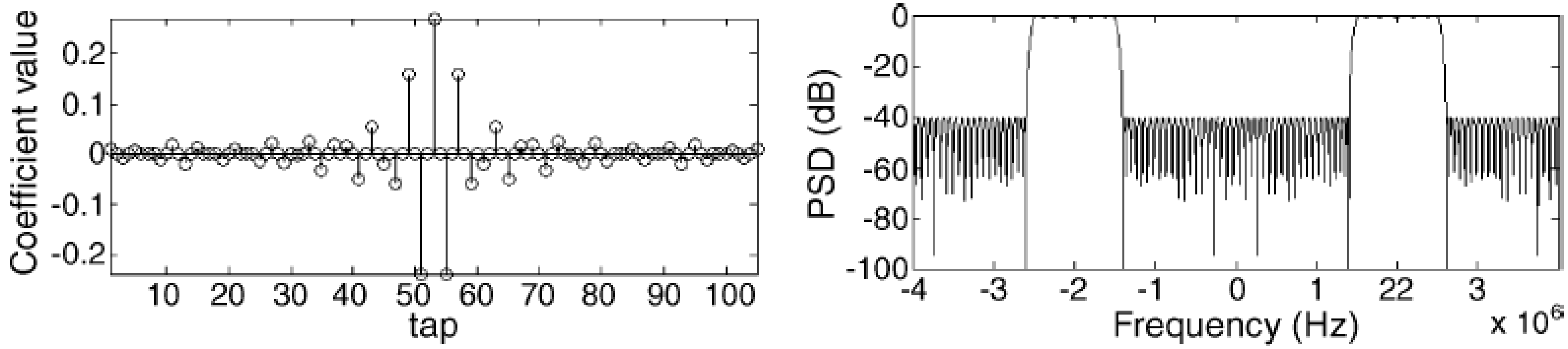
# 7.2.1 Kırpma ve Filtreleme

| Parameters | Value |
|---|---|
| Bandwidth, | 1MHz |
| Sampling frequency, with oversampling factor $BW$ | 8 MHz |
| Carrier frequency, | 2 MHz |
| FFT size, $f_s = BW \cdot L$ $L = 8$ | 128 |
| Number of guard interval samples (CP) $f_c$ | 32 |
| Modulation order $N$ | QPSK |
| Clipping Ratio (CR) | 0.8, 1.0, 1.2, 1.4, 1.6 |

**Table 7.1** Parameters used for simulation of clipping and filtering.

- Program 7.11 “PDF_of_clipped_and_filtered_OFDM_signal.m”
- Program 7.12 “clipping” to clip a signal with CR (Clipping Ratio)

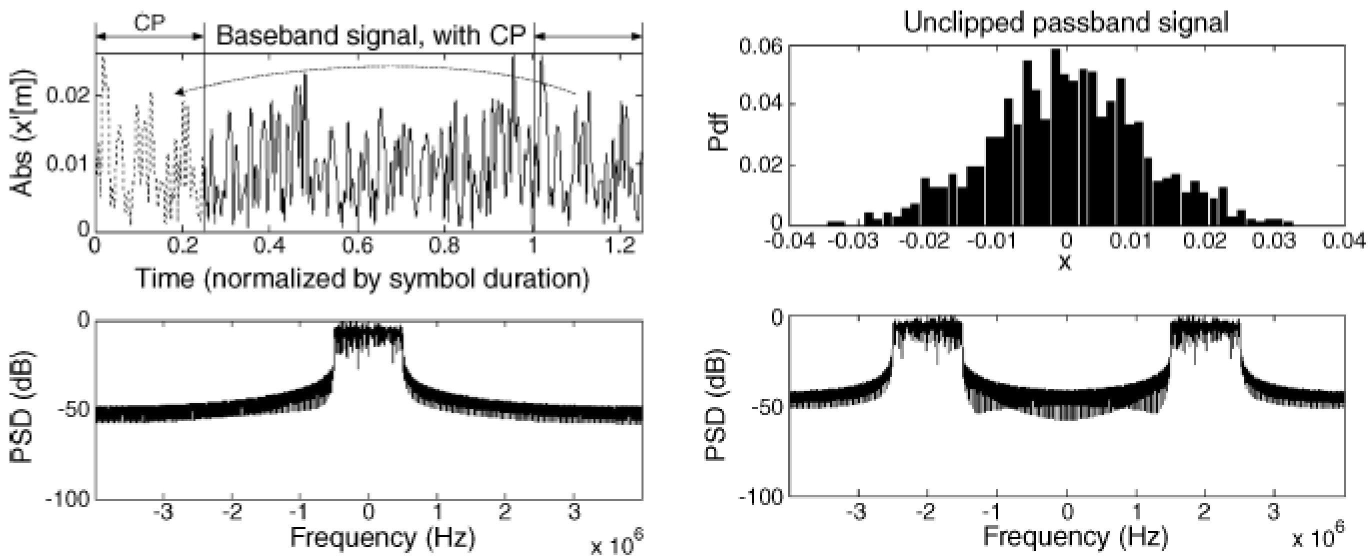
# 7.2.1 Kırpma ve Filtreleme




(a) Filter coefficients (b) Frequency response

**Figure 7.14** Characteristics of an equiripple passband FIR filter.

# 7.2.1 Kırpma ve Filtreleme

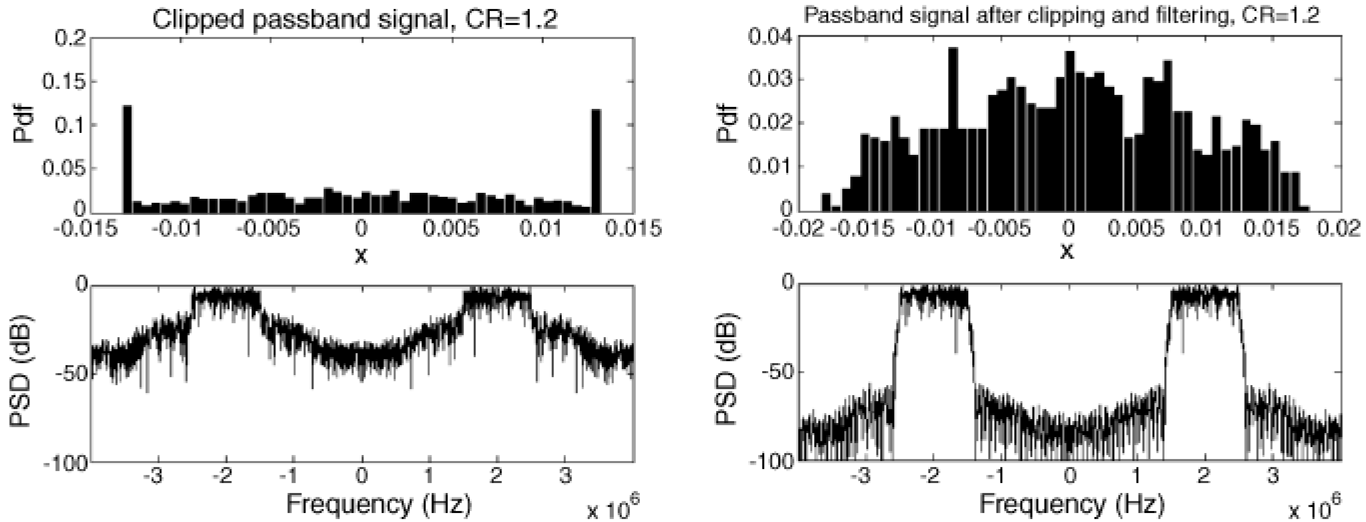



(a) Baseband signal $x'[m]$ and its power spectrum

(b) Pdf and power spectrum of the passband signal $x^p[m]$

**Figure 7.15** Histograms (PDFs) and power spectra of OFDM signals with clipping and filtering (CR = 1.2).

# 7.2.1 Kırpma ve Filtreleme

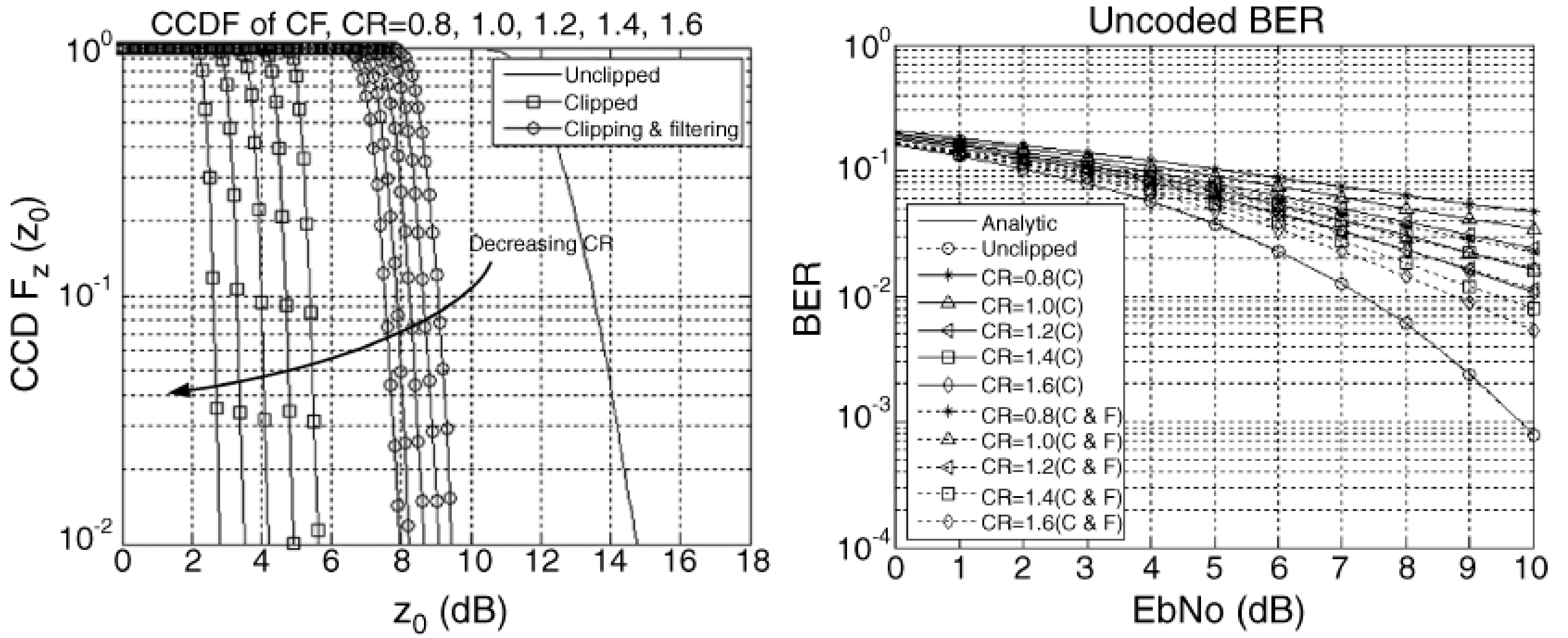



(c) Pdf and power spectrum of the clipped signal $x_c^p[m]$

(d) Pdf and power spectrum of the filtered signal $\tilde{x}_c^p[m]$

**Figure 7.15** Histograms (PDFs) and power spectra of OFDM signals with clipping and filtering (CR = 1.2).

# 7.2.1 Kırpma ve Filtreleme




(a) PAPR distribution (b) BER performance

**Figure 7.16** PAPR distribution and BER performance with clipping and filtering.

# 7.2.2 – 7.2.7 Diğer PAPR Azaltma Yöntemleri

- 7.2.2 PAPR Reduction Code
  - Bilgi blok kodlarla kodlanır , en düşük PAPR sağlayan kod kullanılır
    - Karmaşıklığı fazla
- 7.2.3 Selective Mapping
  - QAM sembolleri faz kaydırma sekanslarıyla çarpılır, en uygun sekans seçilir
    - Çok fft operasyonu gerektirir
- 7.2.4 Partial Transmit Sequence
  - QAM sembolleri alt bloklara ayrıldıktan sonra üstteki her bir alt blok için uygulanır
- 7.2.5 Tone Reservation
  - Bazı altyaşıyıcılar sadece PAPR azaltmak amacıyla kullanılır.
    - Veri hızı kaybı
- 7.2.6 Tone Injection
  - Data QAM sembollerinin üzerine PAPR azaltmak amacıyla semboller *«bindirilir»*
    - *Güç sarfiyatı*
- 7.2.7 DFT Spreading

# 7.2.7 DFT Spreading

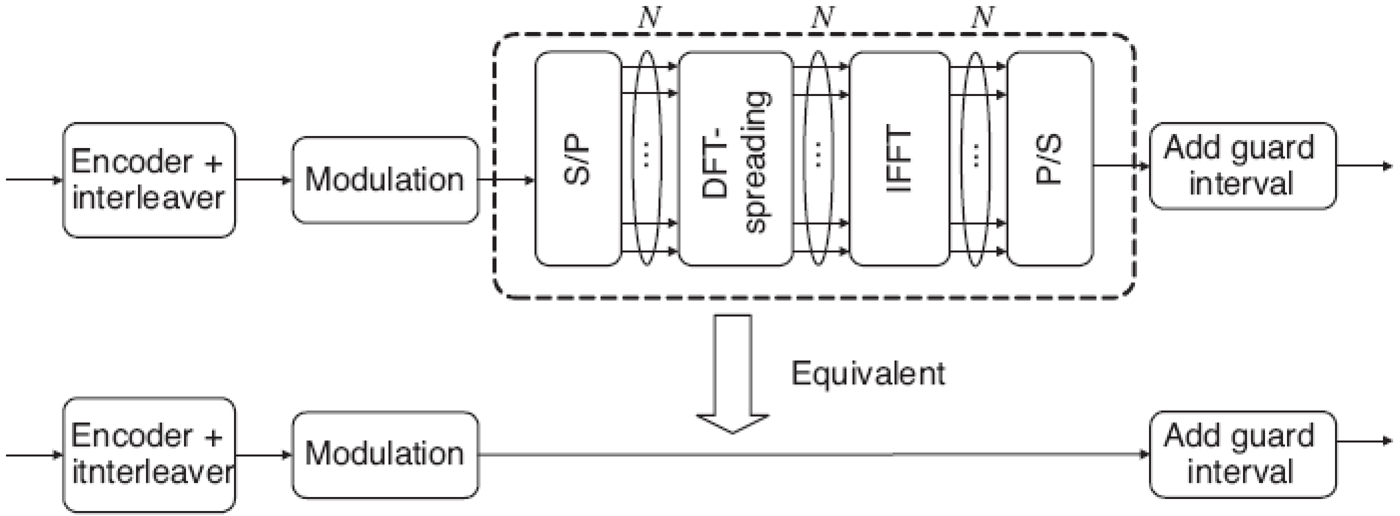



**Figure 7.25** Equivalence of OFDMA system with DFT-spreading code to a single-carrier system.

# 7.2.7 DFT ile yayma

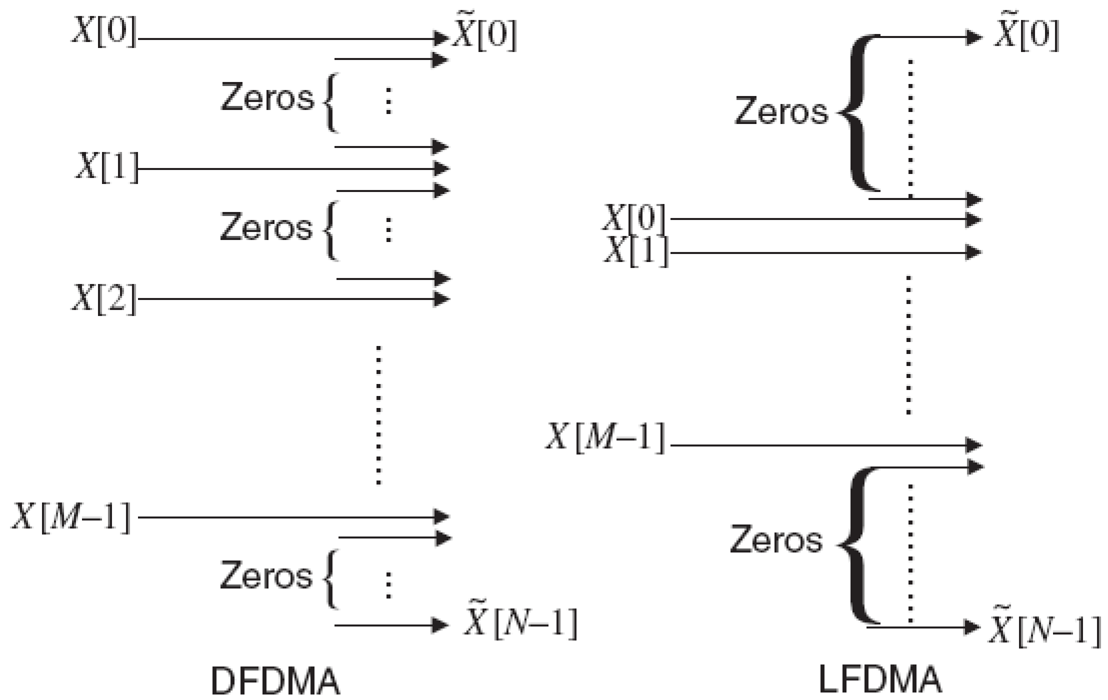



**Figure 7.26** Subcarrier mapping for uplink in OFDMA systems: DFDMA and LFDMA.

# 7.2.7 DFT ile yayma

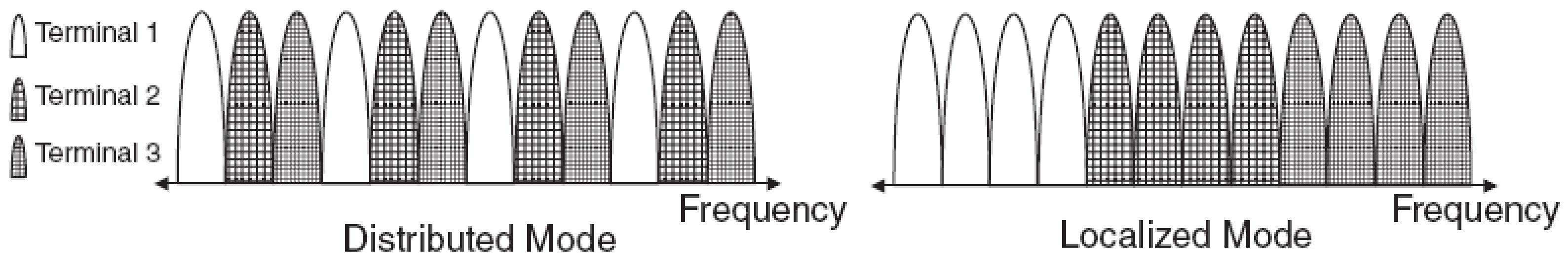



**Figure 7.27** Examples of subcarrier assignment to multiple users: three users with $N = 12$, $M = 4$, and $S = 3$.

# 7.2.7 DFT ile yayma

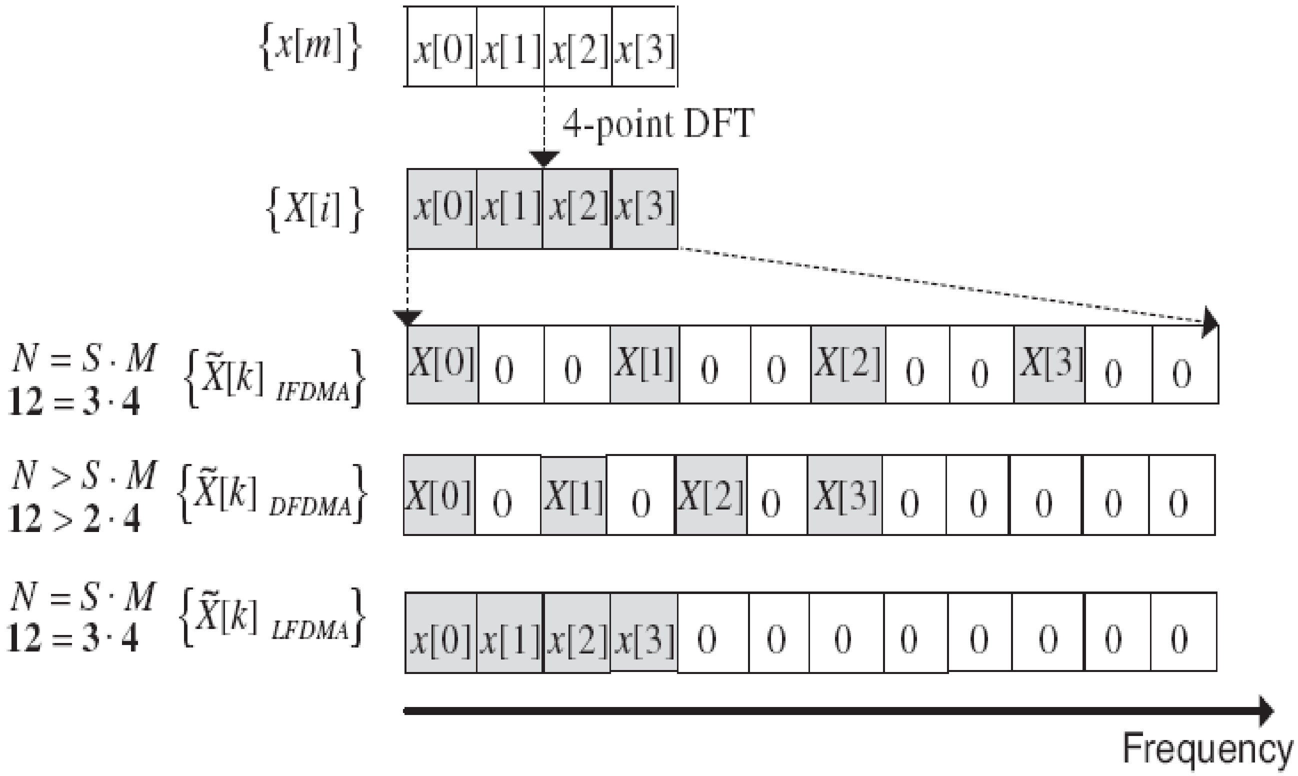



**Figure 7.28** Examples of DFT spreading for IFDMA, DFDMA and LFDMA: three users with $N = 12$, $M = 4$, and $S = 3$.

# 7.2.7 DFT ile Yayma - IFDMA

$$\tilde{X}[k] = \begin{cases} X[k/S], & k = S \cdot m_1, \quad m_1 = 0, 1, 2, \cdots, M-1 \\ 0, & \text{otherwise} \end{cases} \tag{7.35}$$

$$\begin{aligned}
\tilde{x}[n] &= \frac{1}{N}\sum_{k=0}^{N-1}\tilde{X}[k]e^{j2\pi\frac{n}{N}k} \\
&= \frac{1}{S}\cdot\frac{1}{M}\sum_{m_1=0}^{M-1}X[m_1]e^{j2\pi\frac{n}{M}m_1} \\
&= \frac{1}{S}\cdot\frac{1}{M}\sum_{m_1=0}^{M-1}X[m_1]e^{j2\pi\frac{Ms+m}{M}m_1}, \\
&= \frac{1}{S}\cdot\left(\frac{1}{M}\sum_{m_1=0}^{M-1}X[m_1]e^{j2\pi\frac{m}{M}m_1}\right) \\
&= \frac{1}{S}\cdot x[m]
\end{aligned} \tag{7.36}$$

# 7.2.7 DFT Spreading

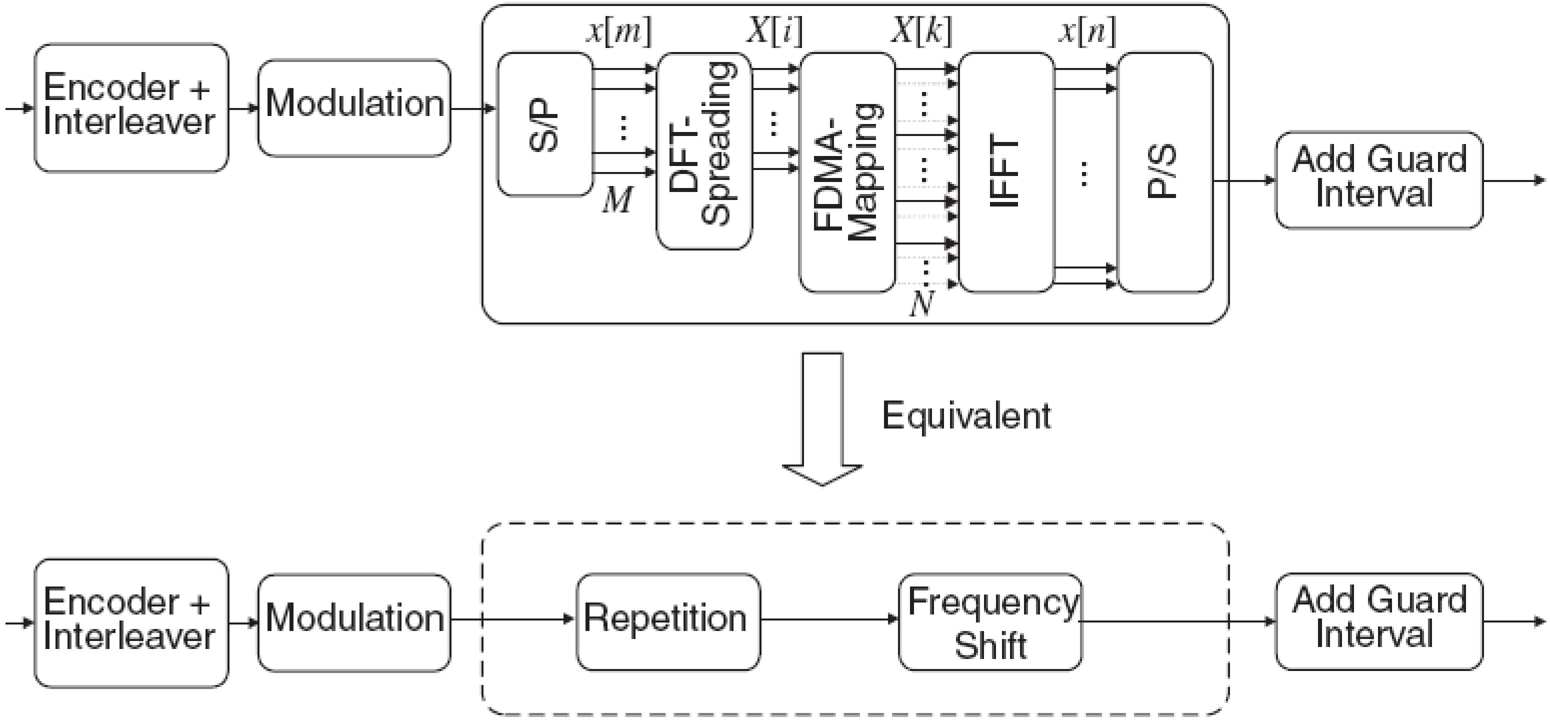



**Figure 7.29** Uplink transmitter with DFT-spreading technique of IFDMA.

# 7.2.7 DFT ile Yayma - IFDMA

$$\tilde{X}[k] = \begin{cases} X[(k-r)/S], & k = S \cdot m_1 + r,\ m_1 = 0, 1, 2, \cdots, M-1 \\ 0, & \text{otherwise} \end{cases} \quad (7.37)$$

$$\begin{aligned}
\tilde{x}[n] &= \tilde{x}[Ms + m] \\
&= \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}[k] e^{j2\pi \frac{n}{N} k} \cdot \\
&= \frac{1}{S} \cdot \frac{1}{M} \sum_{m_1=0}^{M-1} X[m_1] e^{j2\pi \left(\frac{n}{M} m_1 + \frac{n}{N} r\right)} \\
&= \frac{1}{S} \cdot \frac{1}{M} \sum_{m_1=0}^{M-1} X[m_1] e^{j2\pi \frac{(Ms+m)}{M} m_1} e^{j2\pi \frac{n}{N} r} \qquad (7.38) \\
&= \frac{1}{S} \cdot \left( \frac{1}{M} \sum_{m_1=0}^{M-1} X[m_1] e^{j2\pi \frac{m}{M} m_1} \right) \cdot e^{j2\pi \frac{n}{N} r} \\
&= \frac{1}{S} e^{j2\pi \frac{n}{N} r} \cdot x[m]
\end{aligned}$$

# 7.2.7 DFT ile Yayma - LFDMA

$$\tilde{X}[k] = \begin{cases} X[k], & k = 0, 1, 2, \cdots, M-1 \\ 0, & k = M, M+1, \cdots, N-1 \end{cases} \quad (7.39)$$

$$\tilde{x}[n] = \tilde{x}[Sm+s] = \frac{1}{N}\sum_{k=0}^{N-1}\tilde{X}[k]e^{j2\pi\frac{n}{N}k} = \frac{1}{S}\cdot\frac{1}{M}\sum_{k=0}^{M-1}X[k]e^{j2\pi\frac{Sm+s}{SM}k} \quad (7.40)$$

$$\tilde{x}[n] = \tilde{x}[Sm] = \frac{1}{S}\cdot\frac{1}{M}\sum_{k=0}^{M-1}X[k]e^{j2\pi\frac{Sm}{SM}k} = \frac{1}{S}\cdot\frac{1}{M}\sum_{k=0}^{M-1}X[k]e^{j2\pi\frac{m}{M}k} = \frac{1}{S}x[m] \quad (7.41)$$

# 7.2.7 DFT Spreading

$$
\begin{aligned}
\tilde{x}[n] &= \tilde{x}[Sm+s] \\
&= \frac{1}{S}\left(1-e^{j2\pi\frac{s}{S}}\right)\cdot\frac{1}{M}\sum_{p=0}^{M-1}\frac{x[p]}{1-e^{j2\pi\left\{\frac{(m-p)}{M}+\frac{s}{SM}\right\}}} \\
&= \frac{1}{S}e^{j\pi\frac{(M-1)s-Sm}{SM}}\cdot\sum_{p=0}^{M-1}\underbrace{\frac{\sin\left(\pi\frac{s}{S}\right)}{M\sin\left(\pi\cdot\frac{(Sm+s)}{SM}-\pi\frac{p}{M}\right)}}_{\text{Defined as } c(m,s,p)}\cdot\underbrace{\left(e^{j\pi\frac{p}{M}}x[p]\right)}_{\text{Defined as } \hat{x}[p]}
\end{aligned}
\qquad (7.42)
$$

# 7.2.7 DFT Spreading

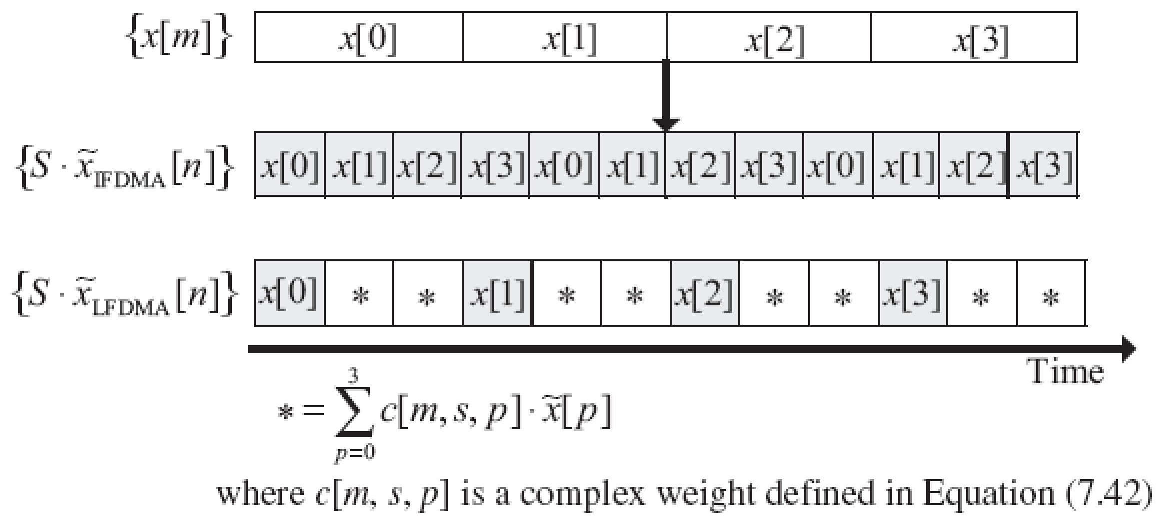



**Figure 7.30** Time-domain signals with DFT-spreading technique: examples for IFDMA and LFDMA.



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# Kodlar

- Program 7.18 (“compare_DFT_spreading.m”),
- Program 7.17 (“CCDF_PAPR_DFTspreading”)
- Program 7.19 “compare_DFT_spreading_w_psf.m”
  - Darbe şekillendirici filtrenin etkisi
- Sonuç
  - Uplink’te kullanılır
  - OFDMA’den daha iyi PAPR performansına sahiptir.

# 7.2.7 DFT Spreading

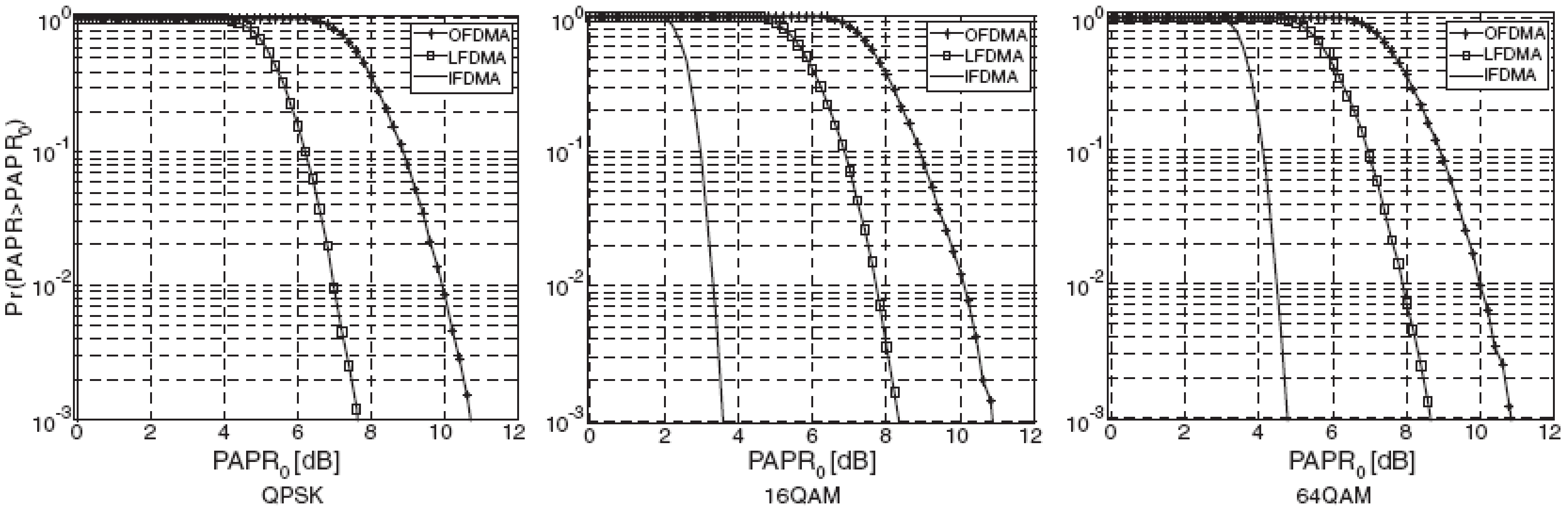


**Figure 7.31** PAPR performances of DFT-spreading technique for IFDMA, LFDMA, and OFDMA.



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# 7.2.7 DFT Spreading

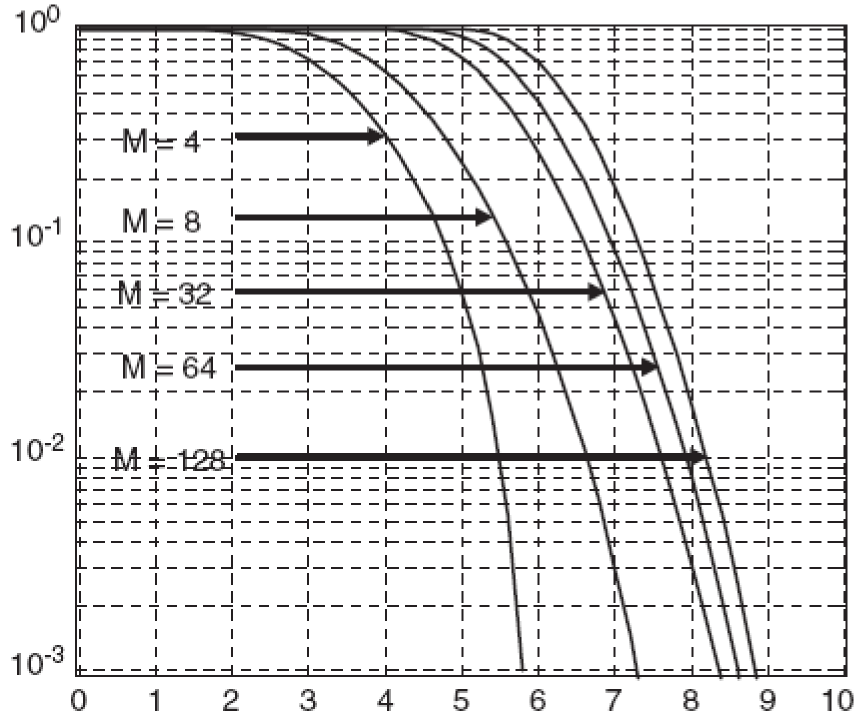


**Figure 7.33** PAPR performance of DFT-spreading technique when $M$ varies.