16.6 UYGULAMALAR

16.6.1 Sistem Kararlılığı (Stability)

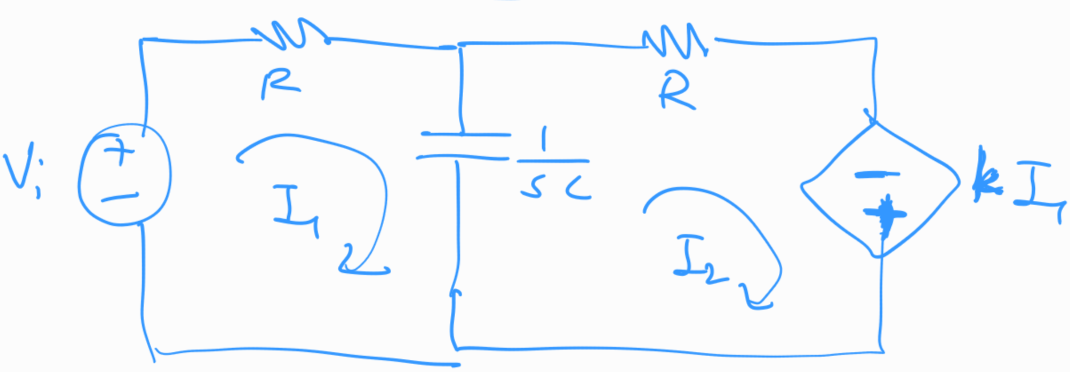
Kararlılık şartı : $\lim\limits_{t \to \infty} |h(t)| < \infty$

$$h(t) = \mathcal{L}^{-1}\{H(s)\}$$

$$H(s) = \frac{N(s)}{D(s)} = \frac{N(s)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$

1. Kararlılık için bütün kutupların gerçel kısımları negatif olmalıdır. Yani karmaşık düzlemin sol tarafında yer almalıdır.
2. Pasif devreler kararsız olamaz.

Exemple 16.13 :



Hangi k değerleri için kararlıdır?

$$-V_i + RI_1 + \frac{1}{sC}(I_1 - I_2) = 0$$

$$\frac{1}{sC}(I_2 - I_1) + RI_2 - kI_1 = 0$$

$$\left(R + \frac{1}{sC}\right) I_1 - \frac{1}{sC} I_2 = V_i$$

$$\left(-\frac{1}{sC} - k\right) I_1 + \left(R + \frac{1}{sC}\right) I_2 = 0$$

$$\underbrace{\begin{bmatrix} R+\frac{1}{sC} & -\frac{1}{sC} \\ -\frac{1}{sC}-k & R+\frac{1}{sC} \end{bmatrix}}_{\bar{A}} \underbrace{\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}}_{\bar{I}} = \underbrace{\begin{bmatrix} V_i \\ 0 \end{bmatrix}}_{\bar{b}}$$

$$\bar{I} = \bar{A}^{-1}\,\bar{b}$$

$$\Delta = \left(R+\frac{1}{sC}\right)^2 - \frac{1}{sC}\left(\frac{1}{sC}+k\right) \neq 0$$ determinant.

karakteristik denklemin

$$R^2 + \frac{2R}{sC} + \cancel{\frac{1}{s^2C^2}} - \cancel{\frac{1}{s^2C^2}} - \frac{k}{sC} = 0$$

$$R^2 = \frac{k-2R}{sC} \Rightarrow \quad s = \frac{k-2R}{R^2C} < 0$$ olmalı

$k < 2R \rightarrow$ kararlılık için.

Example 16.14 :

$$H(s) = \frac{K}{s^2+s(4-k)+1}$$

hangi k değerleri için sistem kararlıdır.

$$s_{1,2} = \frac{-(4-k) \mp \sqrt{(4-k)^2-4}}{2}$$

$-(4-k) < 0$

$4-k > 0$

$\boxed{4 > k \text{ olmalıdır}}$

Problem 16.14

$$H(s) = \frac{1}{s^2+s(25+\alpha)+25}$$

$$s_{1,2} = \frac{-(25+\alpha) \mp \sqrt{(25+\alpha)^2-100}}{2}$$

$-(25+\alpha) < 0$

$25+\alpha > 0$

$\boxed{\alpha > -25}$

16.32 For the network in Fig. 16.55, solve for $i(t)$ for $t > 0$.

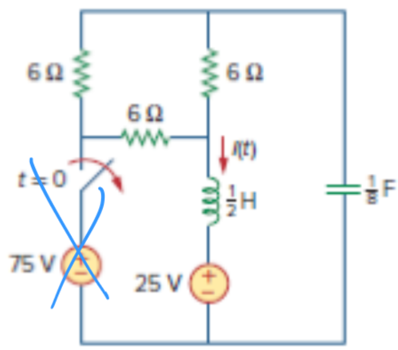
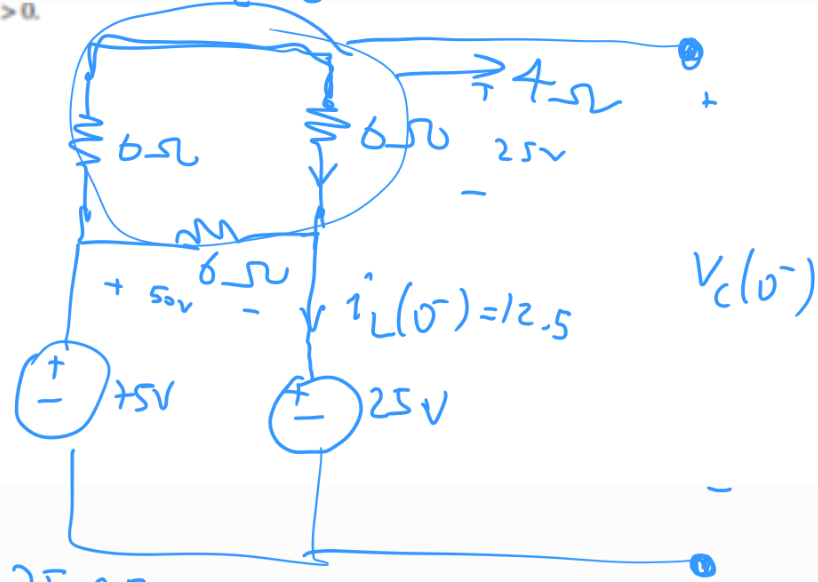


Figure 16.55
For Prob. 16.32.

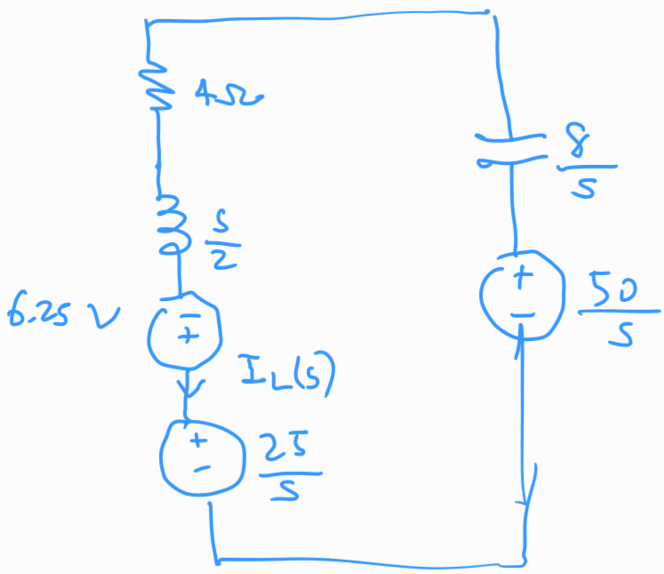
$t = 0^-$



$$\frac{12 \times 6}{18} = 4$$

$$i_L(0^-) = \frac{75 - 25}{4} = 12.5\,A \qquad V_c(0^-) = 50\,V$$

$t > 0$



$$v_L = L\, i_L'$$

$$V_L(s) = sL\, I_L - \underbrace{L\, i_L(0^-)}_{\frac{1}{2} \times 12.5}$$

$$i_c = C\, v_c'$$

$$I_c(s) = sC\, V_c(s) - C\, v_c(0^-)$$

$$\frac{I_c(s) + C\, v_c(0^-)}{sC} = V_c(s)$$

$$\frac{I_c(s)}{sC} + \frac{V_c(0^-)}{s} = V_c(s)$$

$$\frac{25}{s} + 6.25 = I_L(s)\left(\frac{8}{s} + 4 + \frac{s}{2}\right)$$

$$I_L(s) = \frac{\frac{25}{s} + 6.25}{\frac{8}{s} + 4 + \frac{s}{2}}$$

$$= \frac{\frac{25 + 6.25s}{s}}{\frac{16 + 8s + s^2}{2s}} = \frac{50 + 12.5s}{s^2 + 8s + 16} = \frac{12.5(s+4)}{(s+4)^2}$$

$$= \frac{12.5}{s+4}$$

$i_L(t) = 12.5e^{-4t}u(t)$

16.49 Find $i_0(t)$ for $t > 0$ in the circuit in Fig. 16.72.

$\frac{7.5}{s+2}$

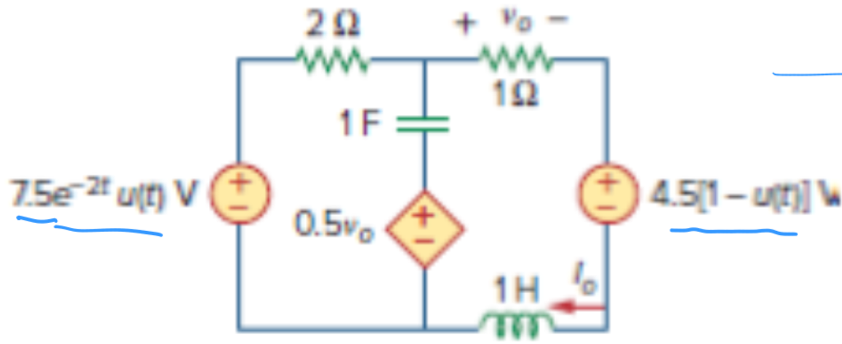
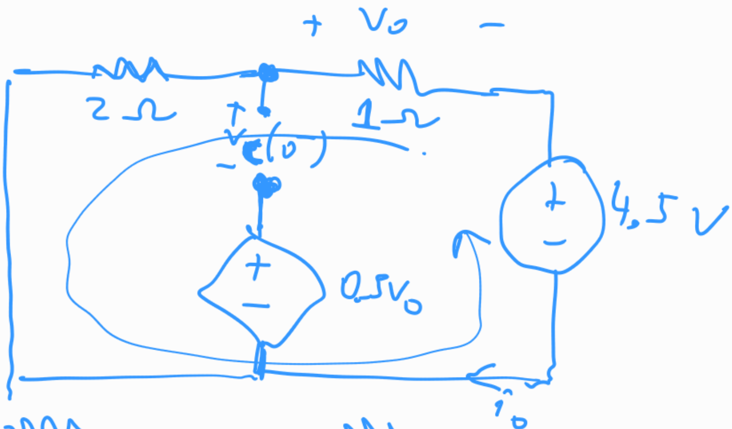


Figure 16.72
For Prob. 16.49.

$t = 0^-$ aninda

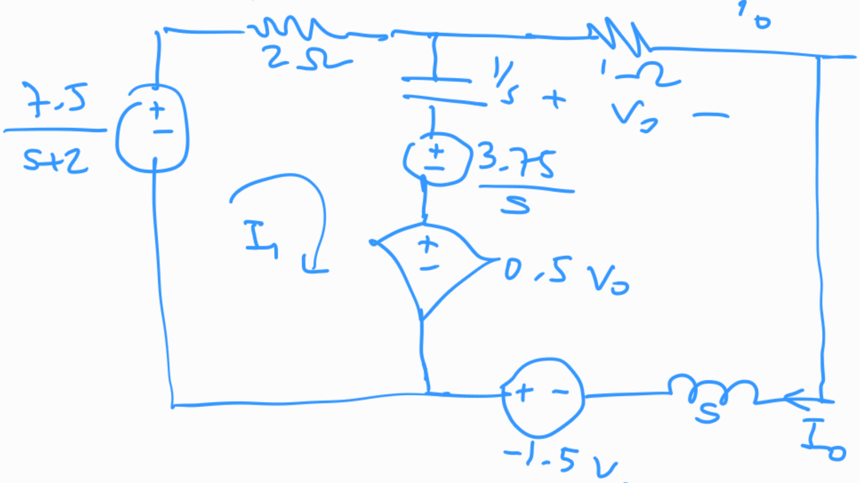


$i_o = \frac{-4.5}{3} = -1.5A = i_L$

$v_o(0^-) = -1.5V$

$0.5v_o = -0.75V$

$v_c(0^-) = 3 - (-0.75)$
$= 3.75$



$V_o = 1 \times I_o$

$$-\frac{7.5}{s+2} + 2I_1 + \frac{1}{s}(I_1 - I_o) + \frac{3.75}{s} + 0.5I_o = 0$$

$$sI_o + 1.5 - 0.5I_o - \frac{3.75}{s} + \frac{1}{s}(I_o - I_1) + I_o = 0$$

$$I_o\left(0.5 - \frac{1}{s}\right) + I_1\left(2 + \frac{1}{s}\right) = \frac{7.5}{s+2} - \frac{3.75}{s} \quad (1)$$

$$I_0\left(s - 0.5 + \frac{1}{s} + 1\right) + I_1\left(-\frac{1}{s}\right) = -1.5 + \frac{3.75}{s} \quad (2)$$

16.54 The switch in Fig. 16.77 has been in position 1 for $t < 0$. At $t = 0$, it is moved from position 1 to the top of the capacitor at $t = 0$. Please note that the switch is a make before break switch; it stays in contact with position 1 until it makes contact with the top of the capacitor and then breaks the contact at position 1. Determine $v(t)$.

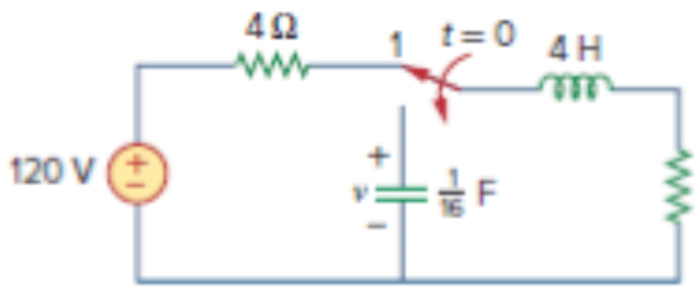
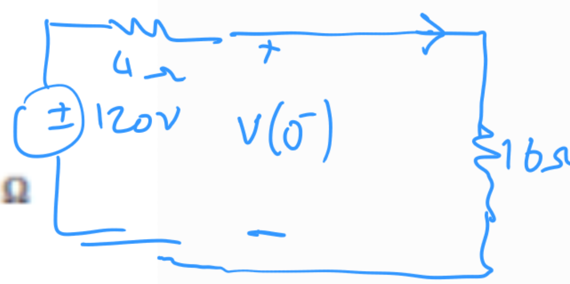


Figure 16.77
For Prob. 16.54.

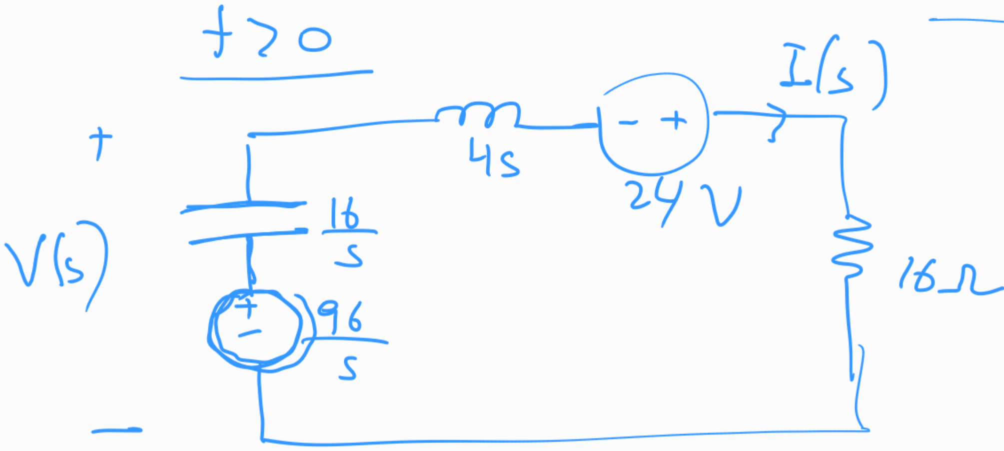
$t = 0^-$



$v(0^-) = 96$ V

$i(0^-) = 6A$

$t > 0$



$$-\frac{96}{s} + I(s)\left(\frac{16}{s} + 4s + 16\right) - 24 = 0$$

$$I(s) = \frac{24 + 96/s}{\frac{16}{s} + 4s + 16} = \frac{\frac{96 + 24s}{\cancel{s}}}{\frac{4s^2 + 16s + 16}{\cancel{s}}}$$

$$\frac{24(s+4)}{4(s^2+4s+4)} = \frac{6(s+4)}{(s+2)^2} = \frac{K_1}{(s+2)^2} + \frac{K_2}{s+2}$$

$$V(s) = \frac{-16}{s} I(s) + \frac{96}{s}$$

$$= \frac{-96(s+4)}{s(s+2)^2} + \frac{96}{s}$$

$$= \frac{K_1}{s} + \frac{K_2}{(s+2)^2} + \frac{K_3}{s+2} + \frac{96}{s} = \cancel{\frac{-96}{s}} + \frac{96}{(s+2)^2} + \frac{96}{s+2} + \cancel{\frac{96}{s}}$$

$$K_1 = \frac{-96 \times 4}{4} = -96$$

$$K_2 = \frac{-96(2)}{-2} = 96$$

$$K_3 = \frac{d}{ds}\left(\frac{-96(s+4)}{s}\right)\Bigg|_{s=-2} = \frac{d}{ds}\left\{-96 - \frac{96 \times 4}{s}\right\}\Bigg|_{s=-2}$$

$$\frac{96 \times 4}{s^2}\Bigg|_{s=-2} = 96$$

$$\boxed{v_o(t) = \left(96te^{-2t} + 96e^{-2t}\right)u(t)}$$

16.71 For the ideal transformer circuit in Fig. 16.94, determine $i_o(t)$.

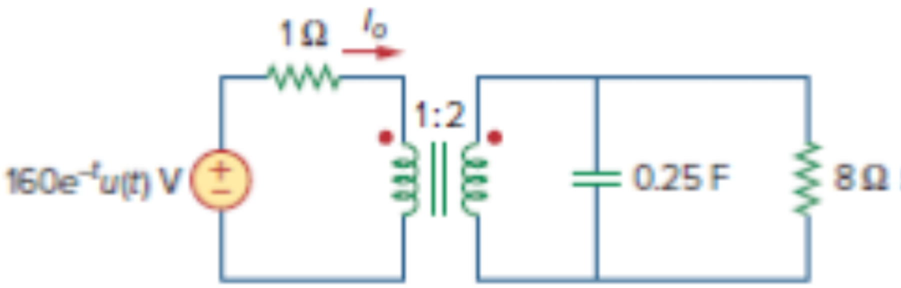
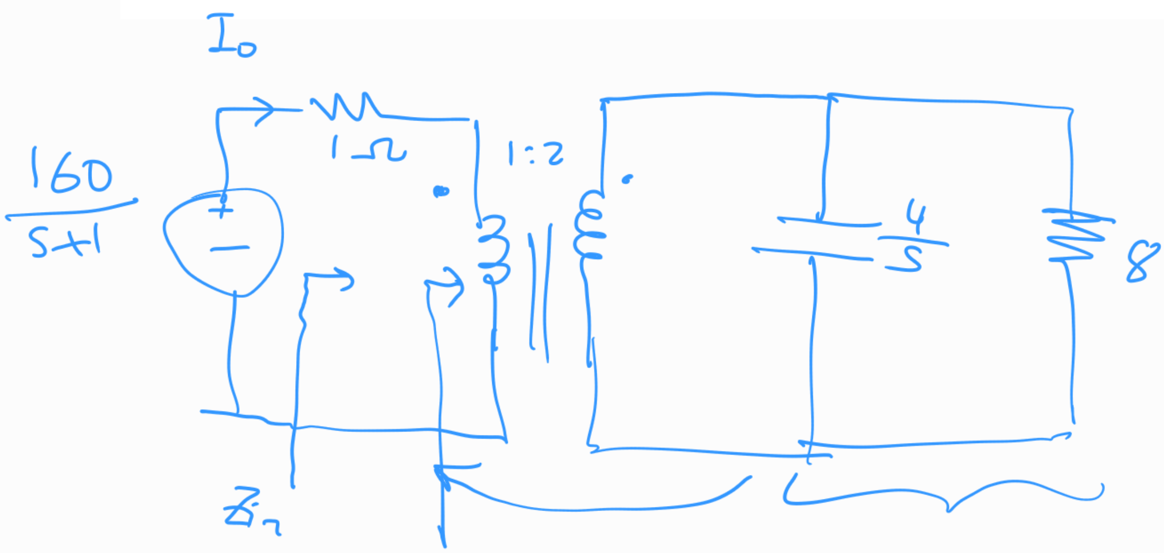
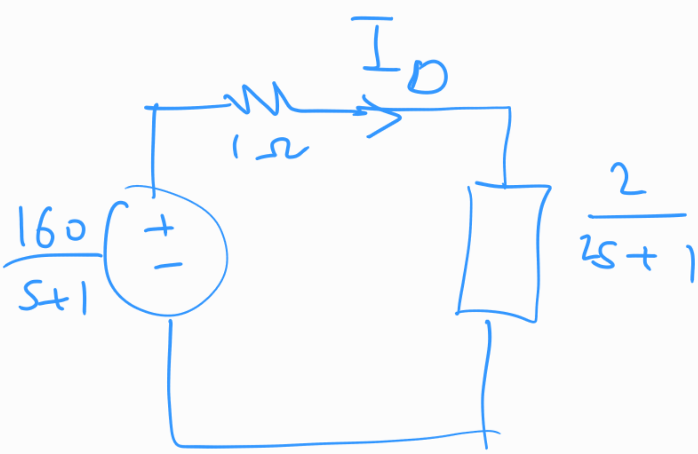


Figure 16.94
For Prob. 16.71.



$$\frac{\frac{32}{s}}{\frac{4}{s}+8} = \frac{\frac{32}{\cancel{s}}}{\frac{4+8s}{\cancel{s}}} = \frac{8}{2s+1}$$



$$\frac{\frac{160}{s+1}}{1+\frac{2}{2s+1}} = \frac{\frac{160}{s+1}}{\frac{2s+3}{2s+1}} = \frac{160(2s+1)}{(s+1)(2s+3)}$$

$$\frac{80(2s+1)}{(s+1)(s+1.5)} = \frac{K_1}{s+1} + \frac{K_2}{s+1.5}$$

$K_1 \to -160$ $\quad$ $K_2 \to 320$

$$K_1 = \frac{80(-1)}{0.5} \qquad K_2 = \frac{80(-2)}{(-0.5)} = 320$$

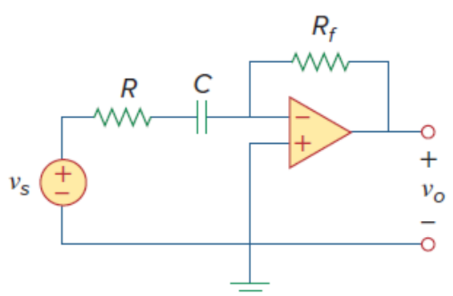
$$\boxed{i_o(t) = \left(-160e^{-t} + 320e^{-1.5t}\right)u(t)}$$

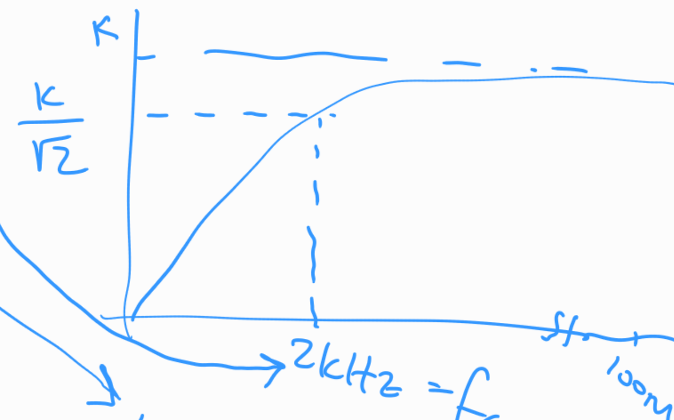
14.69 Design the filter in Fig. 14.94 to meet the following requirements:

(a) It must attenuate a signal at 2 kHz by 3 dB compared with its value at 10 MHz.

(b) It must provide a steady-state output of $v_o(t) = 10\sin(2\pi \times 10^8 t + 180°)$ V for an input $v_s(t) = 4\sin(2\pi \times 10^8 t)$ V.

$10^8 = 100\,MHz$





K = 2.5 olacak

$$V_o = -\frac{R_f}{R} v_s$$

$$\boxed{\frac{R_f}{R} = 2.5 \text{ olacak}}$$

$$\frac{V_s - 0}{R + \frac{1}{sC}} = \frac{-V_o}{R_f}$$

$$\frac{V_o}{V_s} = H(s) = \frac{-R_f}{R + \frac{1}{sC}} = \frac{-R_f\, sC}{1 + sRC}$$

1. derece

$$H(s) = \frac{-\frac{R_f}{R}\, s}{s + \frac{1}{RC}}$$ $\rightarrow \omega_c = 4000\pi$

(1) $4000\pi = \frac{1}{RC}$

$R = 2k\Omega$ desch

(2) $\frac{R_f}{R} = 2.5$

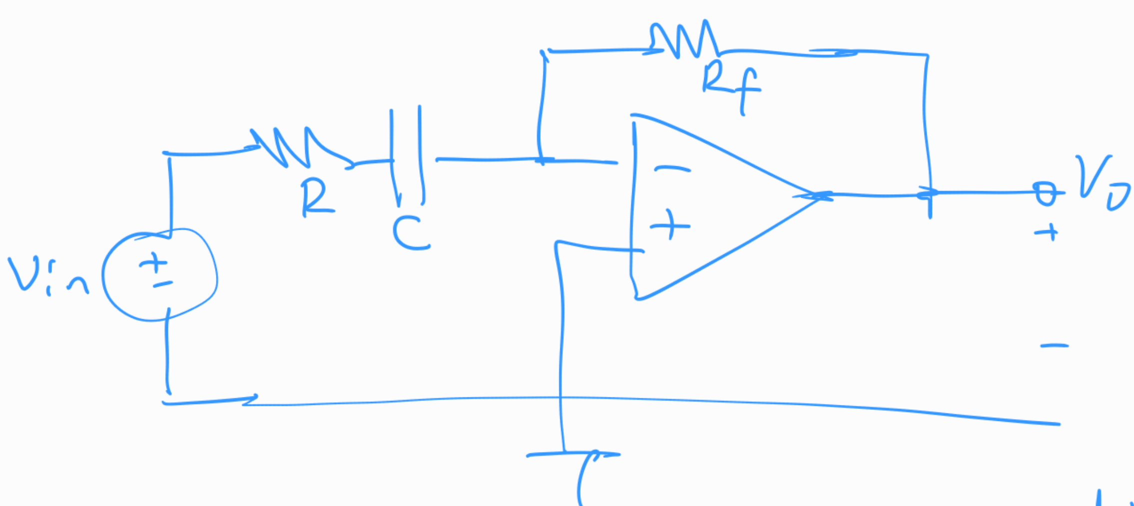
$R_f = 5k\Omega$

$C = \frac{1}{4000\pi \times 2000} = 39.8nF$

14.63 Design an active 1^{st} order HPF with $H(s) = \frac{-100s}{s+10}$ $s = j\omega$

Use a 1-μF capacitor.

$\boxed{K = 100}$ $\boxed{\omega_c = 10\ rad/sn}$



$\frac{R_f}{R} = 100$ $\frac{1}{RC} = 10\ rad/sn$

$C = 1 \times 10^{-6}$ alırsak

$R = 10^{5}\ \Omega$ olur

$R_f = 10 \times 10^{6} = 10\ M\Omega$ olur.