Yüksek Dereceli Filtreler

Sadece kutuplardan oluşan filtre (All-pole filter)

Bu filtrelerin transfer fonksiyonu "Sabit bir sayı bölü polinom" şeklindedir.

Genel ifade : $H(s) = \dfrac{A_{LP}\, a_0}{s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}$

Katsayılar filtre tipine bağlıdır.

En yaygın filtre tipleri Butterworth ve Chebyshev filtreleridir.

$s = j\omega$ yazarsak $|H(\omega)|$ büyüklüğü yüksek frekanslarda yaklaşık olarak $|H(j\omega)| = \dfrac{A_{LP}\, a_0}{\omega^n}$ olur.

Yani Bode çizgesi çizildiğinde bu $-20n$ dB/dekat eğimle iner. Örneğin 6. dereceden bir filtre -120 dB/dekat eğimli Bode çizgesine sahiptir.

Butterworth Düşük geçiren filtresi

$$|H(j\omega)| = \frac{A_{LP}}{\sqrt{1 + \left(\omega/\omega_c\right)^{2n}}} \quad , \quad n = 1, 2, 3, \ldots$$

n yüksek olsun:

$\dfrac{\omega}{\omega_c} < 1$ ise $\longrightarrow |H(j\omega)| \longrightarrow A_{LP}$

$\dfrac{\omega}{\omega_c} > 1$ ise $\longrightarrow |H(j\omega)| \longrightarrow 0$

$n \to \infty$

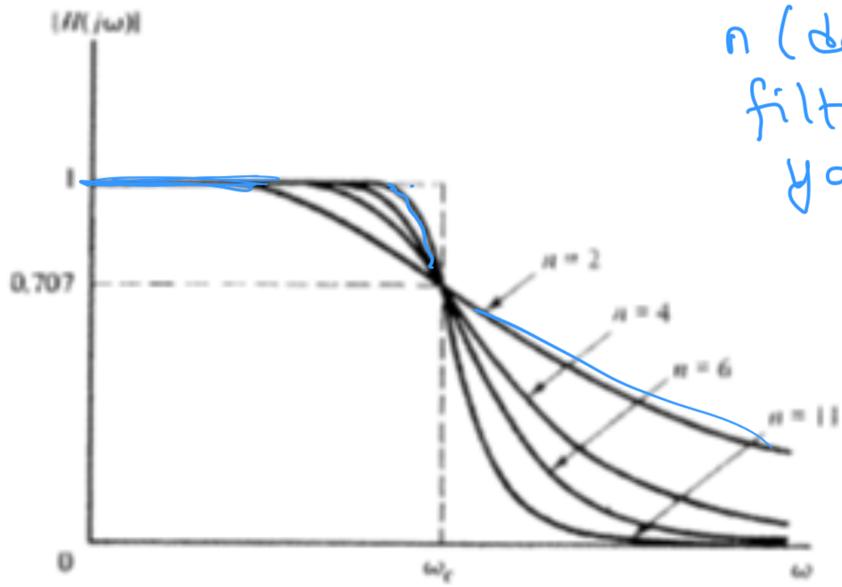


FIGURE 5.5 Low-pass Butterworth magnitudes.

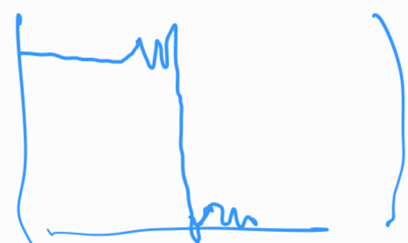
n (derece) arttıkça filtre ideale daha yakın hale gelir.

ω_c : kesim frekansı

A_{LP} : Kazanç.

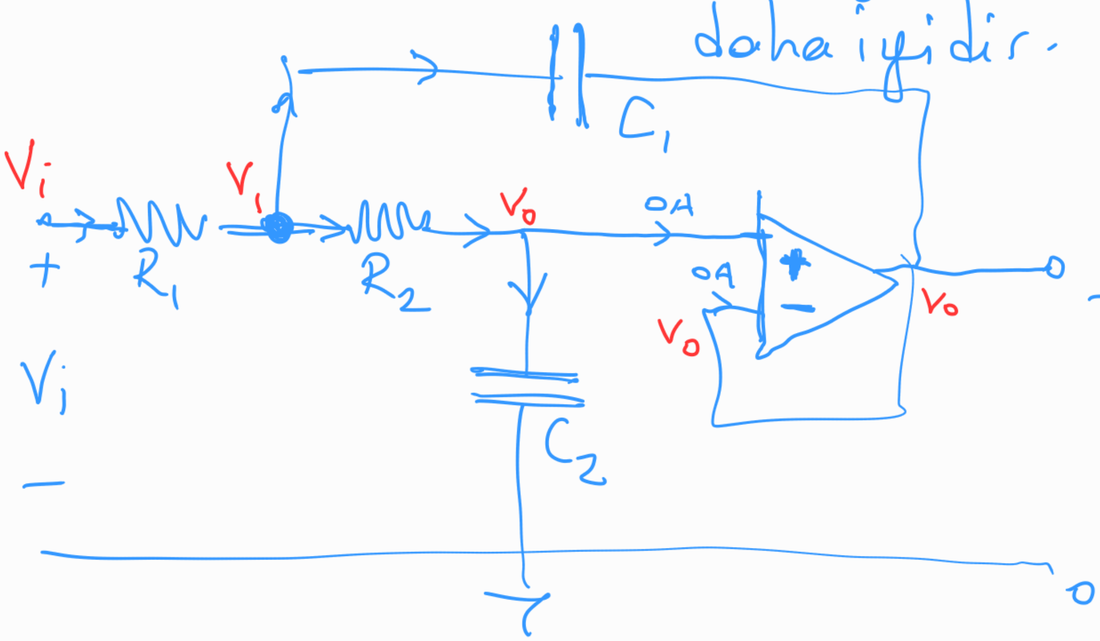
- Butterworth filtresinde $|H(j\omega)|$ ω'ya göre monoton olarak azalır.

(Cheyshev'de öyle değil :



)

- Butterworth filtresi sıfır frekansta en düz olan filtredir — Maximally flat
- Ancak bu filtre yüksek frekanslarda o kadar da iyi değildir. Bu bölgede Chebyshev filtresi daha iyidir.



Transfor fonksiyonu. ?

$\omega \to 0 \quad V_o \to V_i \qquad (C_1, C_2 \text{ açık devre})$

$\omega \to \infty \quad V_o \to 0 \qquad (C_1, C_2 \text{ kısa devre})$

Alçak geçiren filtre.

$$(1)\quad \frac{V_i - V_1}{R_1} = \frac{V_1 - V_o}{R_2} + \frac{V_1 - V_o}{\frac{1}{j\omega C_1}}$$

$$(2)\quad \frac{V_1 - V_o}{R_2} = \frac{V_o}{\frac{1}{j\omega C_2}}$$

$$(1)\quad \frac{V_i}{R_1} = V_1\left(\frac{1}{R_1} + \frac{1}{R_2} + j\omega C_1\right) - V_o\left(\frac{1}{R_2} + j\omega C_1\right)$$

$$(2)\quad \frac{V_1}{R_2} = V_o\left(\frac{1}{R_2} + j\omega C_2\right) \to V_1 = V_o\left(1 + j\omega C_2 R_2\right)$$

$$H(j\omega) = \frac{V_o}{V_i}$$

$$(1)\quad \frac{V_i}{R_1} = V_o\left(1 + j\omega C_2 R_2\right)\left(\frac{1}{R_1} + \frac{1}{R_2} + j\omega C_1\right) - V_o\left(\frac{1}{R_2} + j\omega C_1\right)$$

$$V_i = V_o\left[\left(1 + j\omega C_2 R_2\right)\left(1 + \frac{R_1}{R_2} + j\omega C_1 R_1\right) - \frac{R_1}{R_2} - j\omega C_1 R_1\right]$$

$$V_i = V_o\left[1 + \cancel{\frac{R_1}{R_2}} + \cancel{j\omega C_1 R_1} + j\omega C_2 R_2 + j\omega C_2 R_1 - \omega^2 C_1 C_2 R_1 R_2 - \cancel{\frac{R_1}{R_2}} - \cancel{j\omega C_1 R_1}\right]$$

$$V_i = V_0\left[1 + j\left(wC_2R_2 + wC_2R_1\right) - w^2 C_1C_2R_1R_2\right]$$

$$\frac{V_0}{V_i} = \frac{1}{1 + j\left(wC_2R_2 + wC_2R_1\right) - w^2 C_1C_2R_1R_2} = H(jw)$$

$$H(s) = \frac{1}{1 + sC_2\left(R_1+R_2\right) + s^2 C_1C_2R_1R_2}$$

$$\boxed{H(s) = \frac{1/C_1C_2R_1R_2}{s^2 + s\frac{1}{C_1}\left(\frac{1}{R_2} + \frac{1}{R_1}\right) + \frac{1}{C_1C_2R_1R_2}}}$$

ikinci dereceden bir filtredir.

Genel form (2. derece)

$$H(s) = \frac{K b w_c^2}{s^2 + a w_c s + b w_c^2}$$

Bize kesim frekansı verilirse önce $R_1 = R_2 = 1k\Omega$ alıp, oradan C_1 bulunur.

$$\frac{1}{C_1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = a\underline{w_c}$$

Sonra da

$$b w_c^2 = \frac{1}{C_1C_2R_1R_2}$$ eşitliğine R_1, R_2, C_1 konarak C_2 bulunur, böylece filtreyi tasarlamış oluruz.

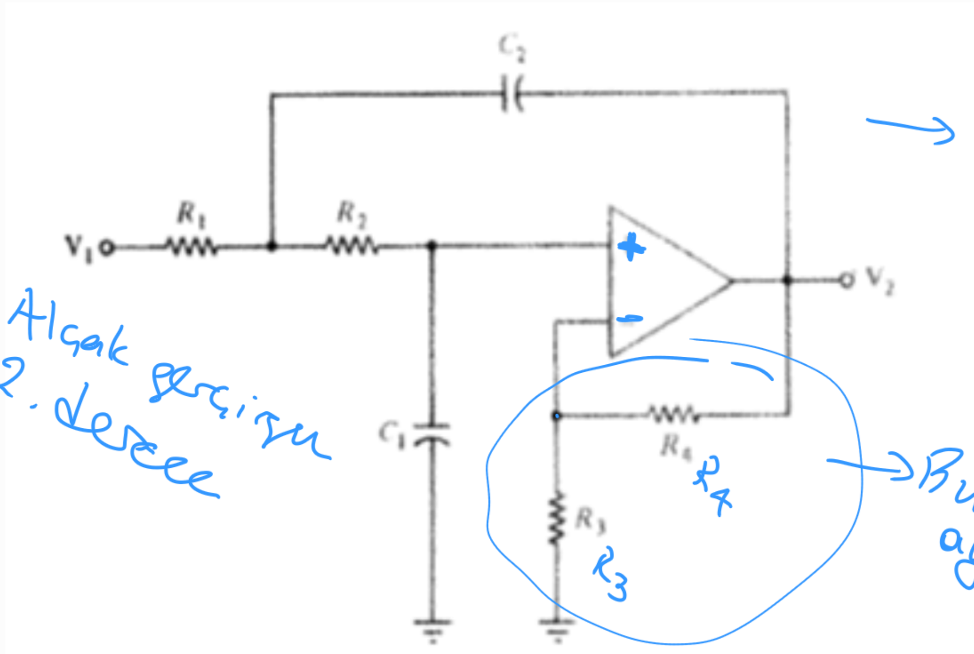


FIGURE 5.12 Second-order VCVS low-pass filter circuit.

→ Önceki devreyle aynı

Alçak geçirgen 2. derece

→ Bu kısım ayrıca kazanç sağlar.

$$K = \frac{R_3 + R_4}{R_3}$$

Butterworth Katsayıları

n	a	b
1	-	1
2	1.414214 → $\sqrt{2}$	1
3	1.000000	1
	-	1
4	0.765367	1
	1.847759	1
5	-	1
	0.618	1
	1.618	1
6		
7		
8		

LPF 2nd order cutoff 3000 Hz
Gain 3

① $\frac{1}{C_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = \underbrace{a}_{\sqrt{2}}\, w_c \rightarrow 6000\pi$

$R_1 = R_2 = 1k\Omega$ alalım.

② $\frac{1}{C_1 C_2 R_1 R_2} = \underbrace{b}_{1=b}\, \omega_c \rightarrow 6000\pi$

① $\frac{1}{C_1}\left(2\times10^{-3}\right) = \sqrt{2}\times 6000\pi$

$$C_1 = \frac{2\times10^{-3}}{\sqrt{2}\times 6000\pi} = 7.5\times10^{-8} = \underline{\underline{75\,nF}}$$

② $\frac{1}{C_1 C_2\, 10^6} = 6000\pi$

$C_1 \to 75\times10^{-9}$

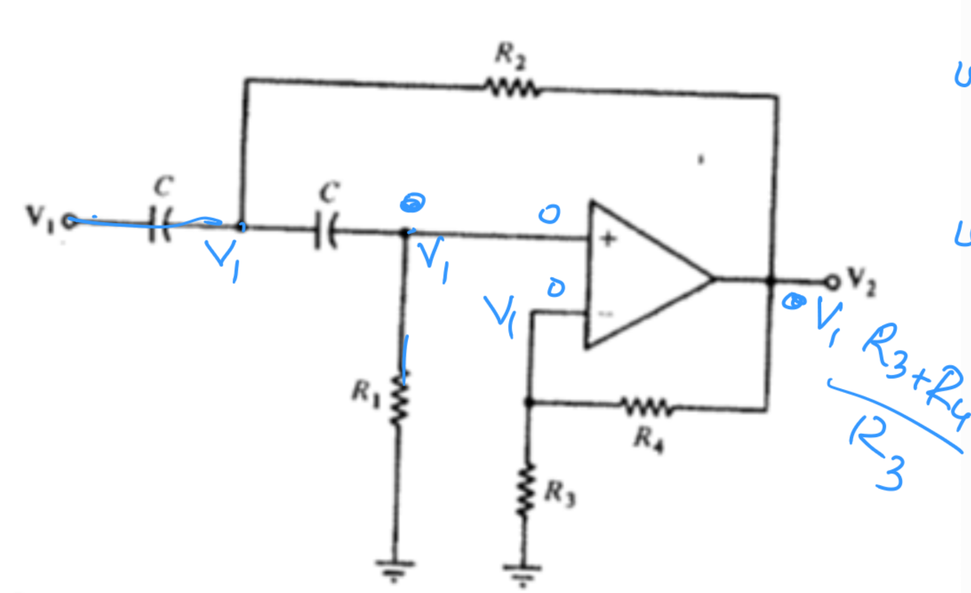


FIGURE 6.4 Second-order VCVS high-pass filter circuit.

$\omega \to 0$: C açık devre

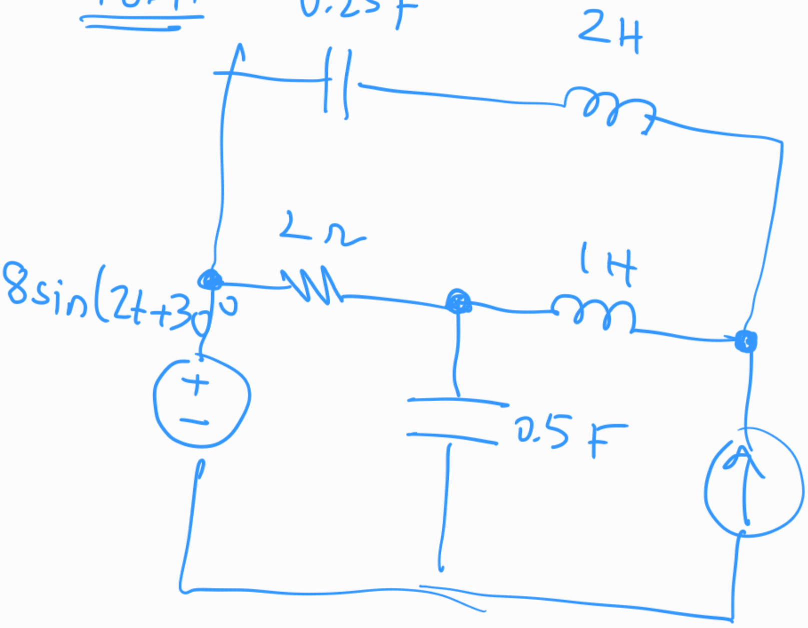
$V_o \to 0$

$\omega \to \infty$: C kısa devre

$V_o \to V_1 \frac{R_3+R_4}{R_3}$

yüksek geçirgen.

$\underline{\underline{10.11}}$



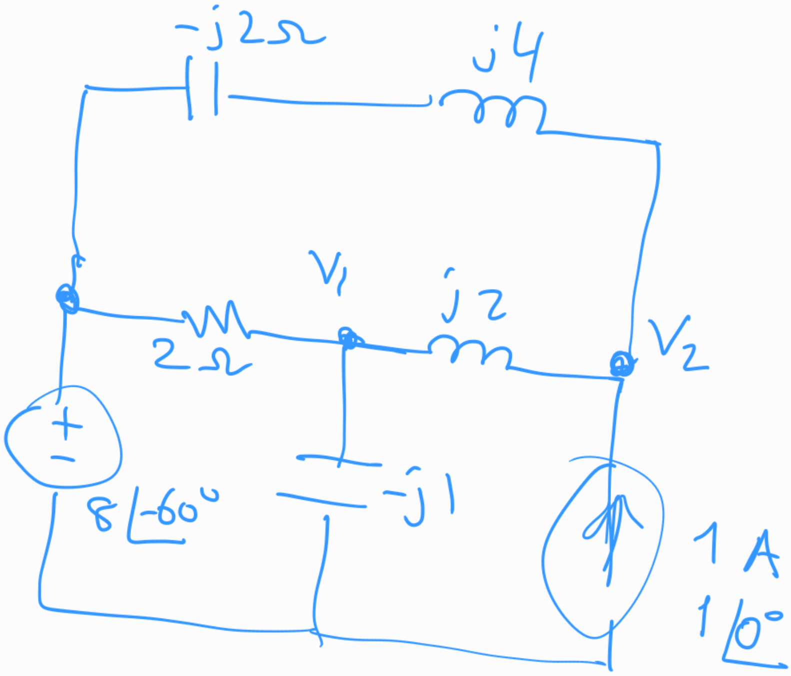
$\omega = 2$

$\frac{-j}{2\times0.25} = -j2$

cos 2t

$8\sin(2t+30°)$

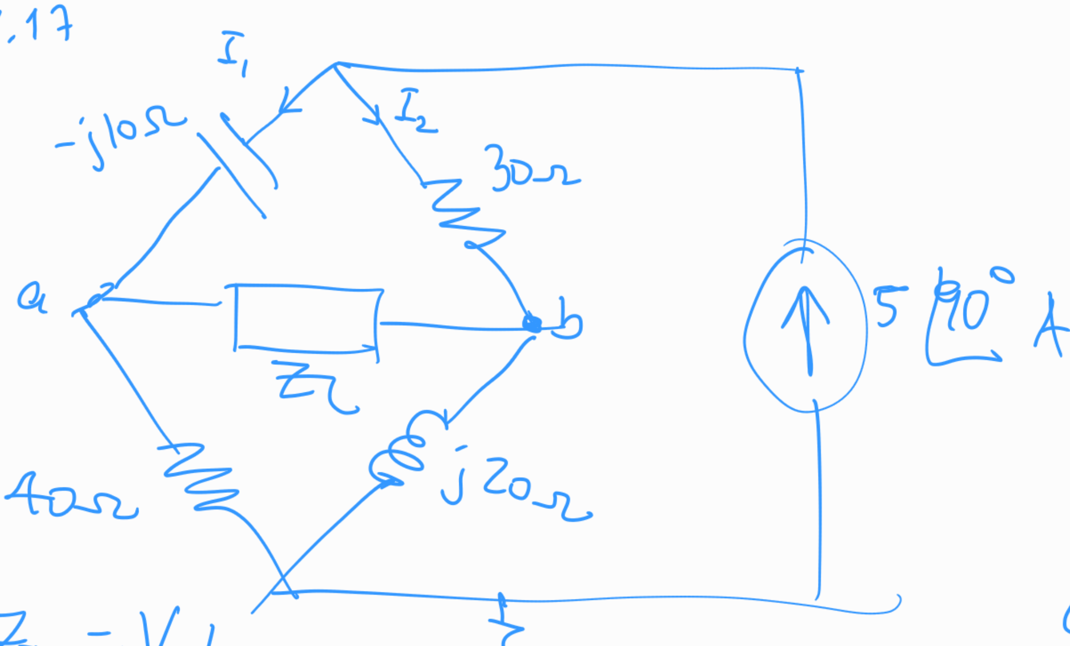
$8\cos(2t+30-90°)$



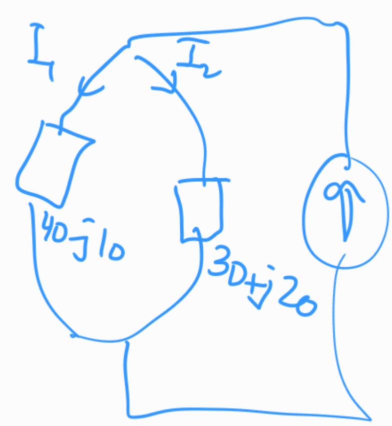
$$\frac{V_1 - 8\angle{-60^\circ}}{2} + \frac{V_1}{-j} + \frac{V_1 - V_2}{j2} = 0 \quad (1)$$

$$\frac{V_2 - V_1}{j2} + \frac{V_1 - 8\angle{-60^\circ}}{4 - j2} = 1 \quad (2)$$

11.17



$$Z_{Th} = \frac{V_{ad}}{I_{kd}}$$



Vad: $I_1 = \frac{5\angle 90^\circ \, (30 + j20)}{70 + j10}$

$$I_2 = \frac{5\angle 90^\circ (40 - j10)}{70 + j10}$$

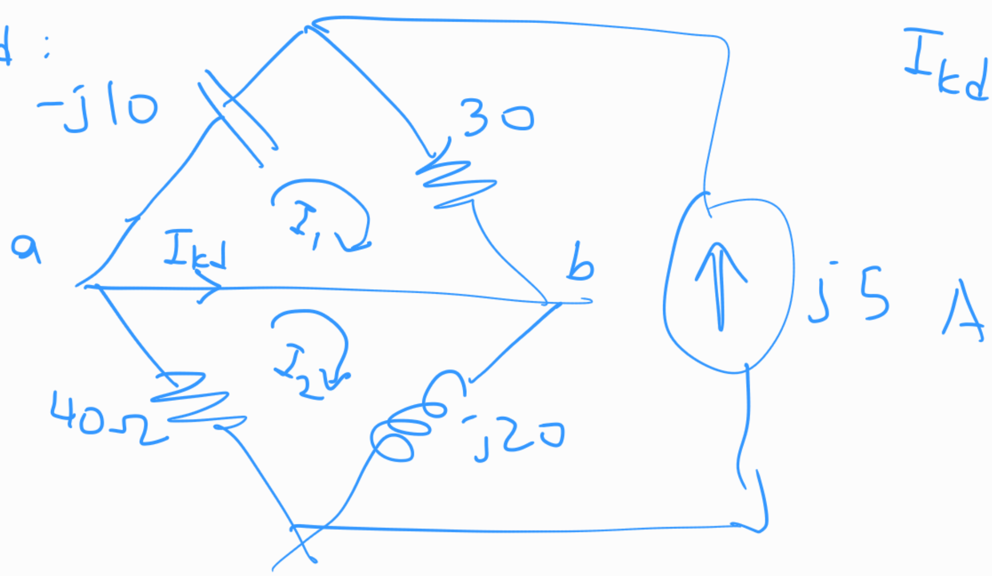
$$V_{ad} = \underset{40 I_1}{\underset{\downarrow}{V_a}} - \underset{j20 I_2}{\underset{\downarrow}{V_b}}$$

$$= \frac{200j(30 + j20)}{70 + j10} + \frac{100(40 - j10)}{70 + j10}$$

$$= \frac{6000j - \cancel{4000} + \cancel{4000} - j1000}{70 + j10}$$

$$\underline{V_{ad} = \frac{5000j}{70 + j10} = 10 + j70}$$

I_{kd} :



$I_{kd} = I_2 - I_1$

$$-j10 I_1 + 30(I_1 + j5) = 0$$

$$j20(I_2 + j5) + 40 I_2 = 0$$

$$I_1 = \frac{-j150}{30 - j10} \qquad I_{kd} = I_2 - I_1$$

$$I_2 = \frac{100}{40 + j20} \qquad = \frac{100}{40 + j20} + \frac{j150}{30 - j10}$$

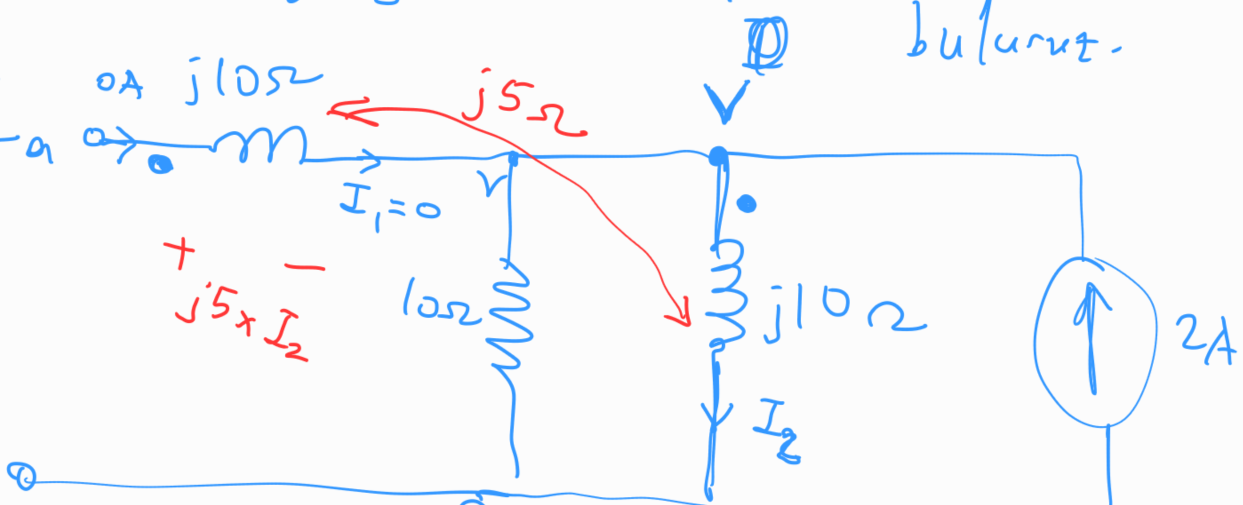
$$\boxed{I_{td} = 0.5 + 3.5i}$$

$$Z_{Th} = \frac{10+70j}{0.5+3.5j} = 20\,\Omega$$

$$(30-j10) \,\|\, (40+j20) = 20\,\Omega$$

→ akım kaynağını sıfırlayarak kolayca buluruz.

13.15



Norton eq. ?

$$\frac{V}{10} + \frac{V}{j10} = 2$$

$$V = \frac{2(10j)}{1+j} = \frac{20j}{1+j}$$

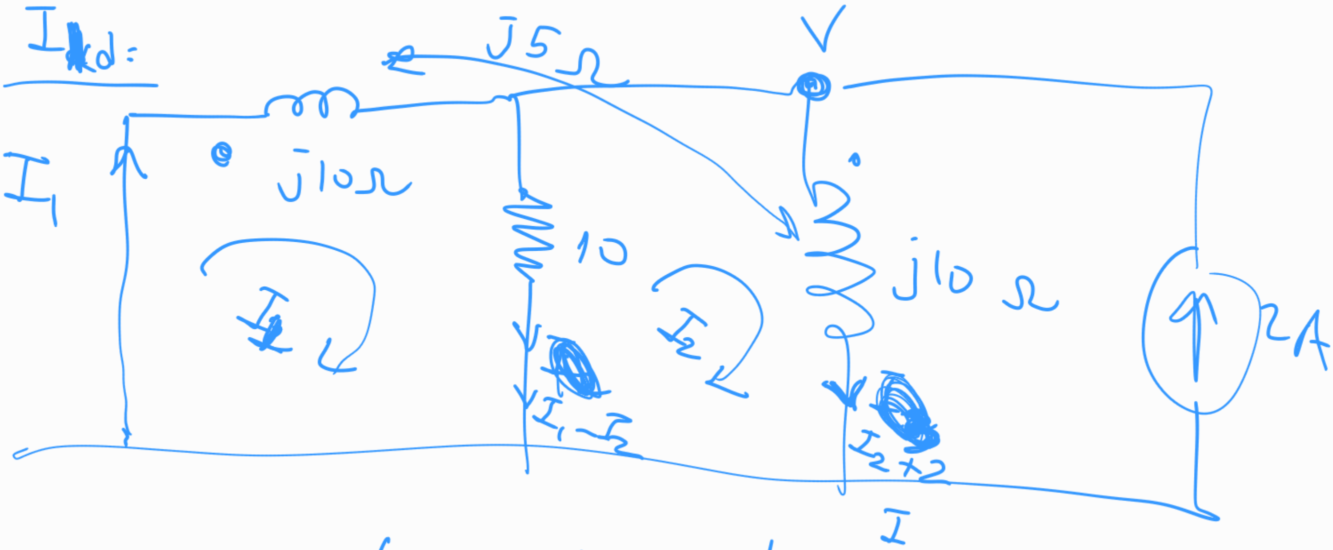
$$I_2 = \frac{20j}{1+j}\,\frac{1}{10j} = \frac{2}{1+j}$$

$$V_{ad} = V + j5 I_2$$

$$V_{ad} = \frac{20j}{1+j} + j5\,\frac{2}{1+j} = \boxed{\frac{30j}{1+j}}$$

I_{kd}:

$I_{kd} = I_1$



$$j10 I_1 + j5(I_2 + 2) + 10(I_1 - I_2) = 0 \qquad (1)$$

$$10(I_1 - I_2) = j10(I_2 + 2) + j5 I_1 \qquad (2)$$

$I_1 = I_{kd}$

$$\boxed{\begin{aligned} (10 + j10) I_1 + (-10 + j5) I_2 &= -j10 \\ (10 - j5) I_1 + (-10 - j10) I_2 &= j20 \end{aligned}}$$