# 13.4 Doğrusal Trafolar

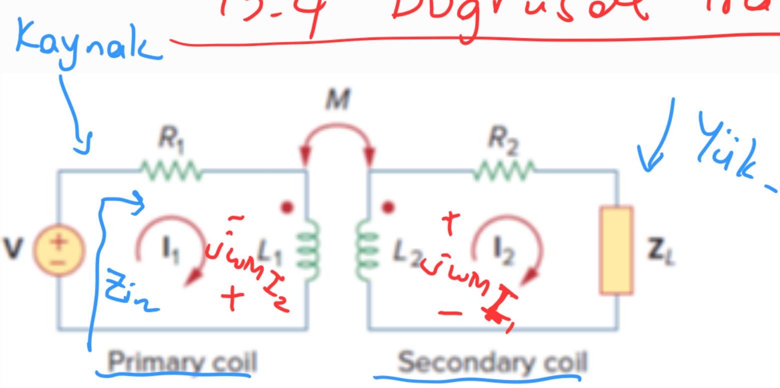
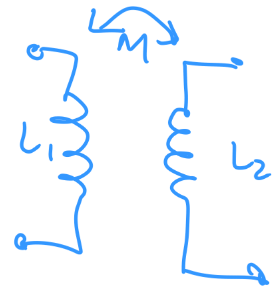



**Figure 13.19**
A linear transformer.




- Trafo eş endüktans kavramından faydalanan, dört terminalli en az iki bobin içeren cihazlardır.
- Gerilim kaynağı → birincil sarım
- Yük → ikincil sarım.
- Dirençler → olası güç kayıplarını modeller.
- Doğrusal trafo: bobinler hava, plastik, bakalit, ahşap gibi malzemeler etrafına sarılı.
- Radyo, TV gibi radyo frekans devrelerinde kullanılır.

Kaynaktan görülen empedansı bulalım: $\rightarrow Z_{in} = \dfrac{\bar{V}}{\bar{I}_1}$

(1) $$-\bar{V} + \bar{I}_1 R_1 + \bar{I}_1 j\omega L_1 - j\omega M \bar{I}_2 = 0$$

(2) $$R_2 \bar{I}_2 + \bar{Z}_L \bar{I}_2 + j\omega L_2 \bar{I}_2 - j\omega M \bar{I}_1 = 0$$

(2) $$\bar{I}_2 (R_2 + j\omega L_2 + Z_L) = j\omega M \bar{I}_1$$

$$\bar{I}_2 = \frac{j\omega M}{R_2 + j\omega L_2 + Z_L} \bar{I}_1$$

(1) $$\bar{V} = \bar{I}_1 (R_1 + j\omega L_1) - \frac{(j\omega M)(j\omega M)\bar{I}_1}{R_2 + j\omega L_2 + Z_L}$$

$$\bar{V} = \bar{I}_1 \left( R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L} \right)$$

$$Z_{in} = \frac{\bar{V}}{\bar{I}_1} = \underbrace{R_1 + j\omega L_1}_{\text{Birincil empedans}} + \underbrace{\frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}}_{\text{Yansıyan (Reflected) Empedans}}$$

Önemli: Yansıyan empedans noktaların konumundan bağımsız olarak hep + işaretli.

**Practice Problem 13.4**

Find the input impedance of the circuit in Fig. 13.25 and the current from the voltage source.

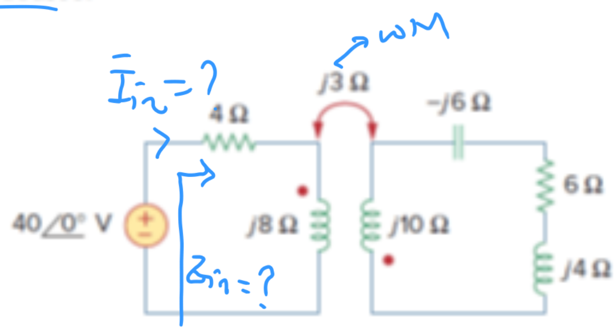



**Figure 13.25**
For Practice Prob. 13.4.

**Answer:** 8.58/58.05° Ω, 4.662/− 58.05° A.

$\omega M = 3$

$$Z_{in} = 4 + j8 + \frac{9}{j10 - j6 + 6 + j4}$$

$$= 4 + j8 + \frac{9}{6 + j8}$$

$$Z_{in} = 4 + j8 + \frac{9}{10\angle 53^\circ} = 4 + j8 + 0.9\angle -53^\circ$$

$$Z_{in} = 4 + j8 + 0.54 - j0.72$$

$$= 4.54 + j7.28 = 8.58\angle 58.05^\circ\ \Omega$$

$$\bar{I}_{in} = \frac{40\angle 0^\circ}{Z_{in}} = \frac{40\angle 0^\circ}{8.58\angle 58.05^\circ} = 4.662\angle -58.05^\circ\ A$$

**Example 13.4**

In the circuit of Fig. 13.24, calculate the input impedance and current $\mathbf{I}_1$. Take $\mathbf{Z}_1 = 60 - j100\ \Omega$, $\mathbf{Z}_2 = 30 + j40\ \Omega$, and $\mathbf{Z}_L = 80 + j60\ \Omega$.

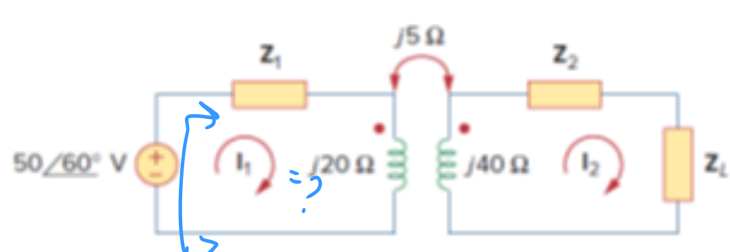



**Figure 13.24**
For Example 13.4.

$$Z_{in} = 60 - j100 + j20 + \frac{25}{j40 + 30 + j40 + 80 + j60}$$

$$= 60 - j80 + \frac{25}{110 + j140} = \underbrace{60 - j80}_{\text{Birincil}} + \underbrace{\frac{5}{22 + j28}}_{\text{Yansıyan}}$$

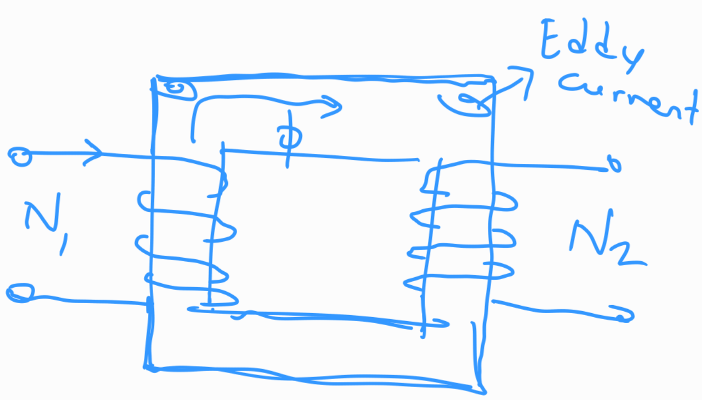
$$Z_{in} = 100.14 \angle -53.1^\circ \ \Omega \qquad \bar{I}_1 = \frac{50\angle 60^\circ}{Z_{in}} = 0{,}5 \angle 113.1^\circ \ A$$

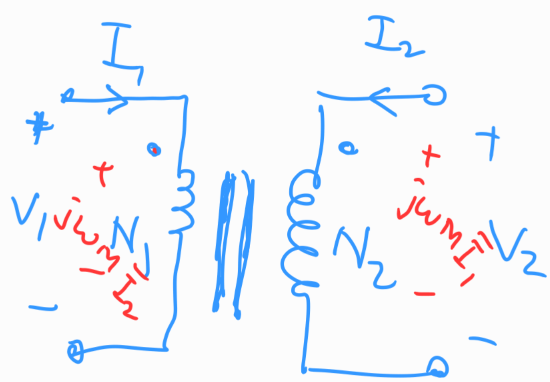
## 13.5 İdeal Trafolar:

$$M = k\sqrt{L_1 L_2} \qquad k \simeq 1$$

Kayıpsız, sonsuz endüktanslı bobinler ve yüksek iletken malzeme

Demir çekirdek kullanılır ve güç sistemlerinde kullanım alanı bulur.




$$① \quad \bar{V}_1 = j\omega L_1 \bar{I}_1 + j\omega M \bar{I}_2$$

$$② \quad \bar{V}_2 = j\omega L_2 \bar{I}_2 + j\omega M \bar{I}_1$$

$$① \quad \bar{I}_1 = \frac{\bar{V}_1 - j\omega M \bar{I}_2}{j\omega L_1}$$

$\boxed{M = \sqrt{L_1 L_2}}$ olursa

(2) $\overline{V_2} = j\omega L_2 \overline{I_2} + j\omega\sqrt{L_1L_2}\left(\dfrac{\overline{V_1} - j\omega\sqrt{L_1L_2}\,\overline{I_2}}{j\omega L_1}\right)$

$$\overline{V_2} = \cancel{j\omega L_2 \overline{I_2}} + \sqrt{\frac{L_2}{L_1}}\,\overline{V_1} - \cancel{j\omega L_2 \overline{I_2}} = \sqrt{\frac{L_2}{L_1}}\,\overline{V_1}$$

$$\boxed{\sqrt{\frac{L_2}{L_1}} = \frac{N_2}{N_1} = n}$$

$$* \boxed{\overline{V_2} = n\,\overline{V_1}}$$

$$\frac{N_2}{N_1} = 10 \qquad \frac{\overline{V_2}}{\overline{V_1}} = 10$$

*Trafoda kayıp olmadığından ve enerji korunumundan,

$$v_1 i_1 = v_2 i_2$$

$$* \boxed{\frac{\overline{V_2}}{\overline{V_1}} = \frac{\overline{I_1}}{\overline{I_2}}} = n$$

$n=1$ : İzolasyon trafosu (elektriksel temas olmadan gerilimi karşı tarafa geçirmek)

$n>1$ : Step-up trafo

$n<1$ : step-down trafo

Yukarıdaki birincil- ikincil ilişkisi, noktaların konumu, ve akım/gerilim tanımlarına göre değişir.

<u>Nokta kuralı:</u>

1) Hem $V_1$ hem de $V_2$ noktalı tarafta pozitif ise negatif gerilim ilişkisi pozitif.

2) Hem $I_1$ hem de $I_2$ noktalı taraftan giriyorsa veya çıkıyorsa akım ilişkisi negatif

Gücün korunumu: $\overline{S}_1 = \overline{V}_1 \overline{I}_1^* = \dfrac{\overline{V}_2}{n} (n \overline{I}_2^*)$

$= \overline{V}_2 \overline{I}_2^* = \overline{S}_2$

Kaynaktan görülen empedans:

$$\overline{Z}_{in} = \frac{\overline{V}_1}{\overline{I}_1} = \frac{\overline{V}_2/n}{\overline{I}_2\, n} = \frac{\overline{V}_2}{\overline{I}_2} \frac{1}{n^2} = \frac{Z_L}{n^2}$$

Yük empedansı kaynakta farklı görünür (empedans uyumlama)

Problem 13.7: 2200/110V trafo (ideal)

$\overline{I}_1 = 25A$

a) turns ratio (n) b) kVA rating c) $\overline{I}_2 = ?$

a) $n = \dfrac{110}{2200} = \dfrac{1}{20}$ (step down)

b) $S = 2200 \times 25 = 55\,kVA$

c) $\overline{I}_2 = \dfrac{55000}{110} = 500A$

**Example 13.8** For the ideal transformer circuit of Fig. 13.37, find: (a) the source current $I_1$, (b) the output voltage $V_o$, and (c) the complex power supplied by the source.

↳ $20 I_2$

4 bilinmeyen

4 denklem

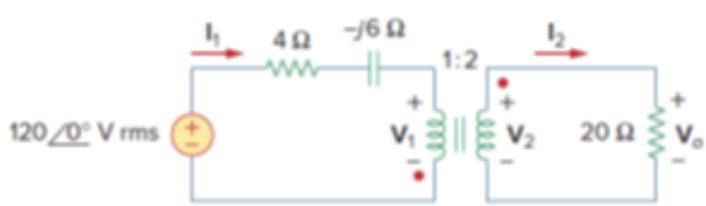



**Figure 13.37**
For Example 13.8.

(1) $-120 + (4-j6) I_1 + V_1 = 0$

(2) $-V_2 + 20 I_2 = 0$

(3) $\dfrac{V_2}{V_1} = -2 \rightarrow V_2 = -2V_1$

(4) $\dfrac{I_2}{I_1} = -\dfrac{1}{2}$

(2) ve (3) $\quad 2V_1 = -20 I_2$

$$\boxed{V_1 = -10 I_2}$$

(1) $\quad 120 = (4-j6)I_1 - 10 I_2 \rightarrow 120 = -2(4-j6) I_2 - 10 I_2$

(4) $\quad I_1 = -2 I_2 \rightarrow 120 = (-18+j12) I_2$

$$I_2 = \frac{120}{-18+j12}$$

$$I_1 = \frac{120}{9-j6}$$

$$V_o = 20 I_2 = \frac{2400}{-18+j12} = \frac{400}{-3+j2} = -92.3 - j61.5$$

$$= 110.9 \angle 213.69^\circ$$

In the ideal transformer circuit of Fig. 13.38, find $\mathbf{V}_o$ and the complex power supplied by the source.

## Practice Problem 13.8

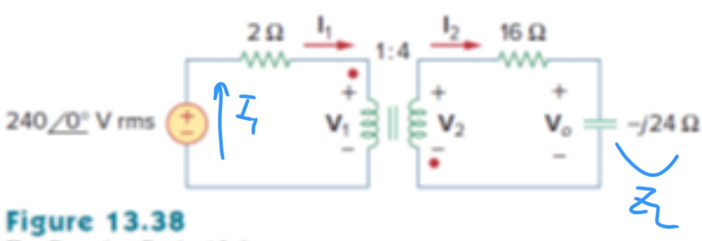



**Figure 13.38**
For Practice Prob. 13.8.

$$\boxed{\overline{V}_o = -j24\, \overline{I}_2}$$

?

$$\boxed{\overline{S} = -240\angle 0^\circ \cdot \overline{I}_1^{*}}$$

?

**Answer:** $429.4\angle 116.57^\circ$ V, $17.174\angle -26.57^\circ$ kVA.

(1) $\quad -240 + 2\overline{I}_1 + \overline{V}_1 = 0$

(2) $\quad -\overline{V}_2 + (16-j24)\overline{I}_2 = 0 \rightarrow \overline{V}_2 = (16-j24)\overline{I}_2$

(3) $\quad \dfrac{\overline{V}_2}{\overline{V}_1} = -4 \Rightarrow \overline{V}_2 = -4\overline{V}_1$

$$-4\overline{V}_1 = (16-j24)\overline{I}_2$$

$$-\overline{V}_1 = (4-j6)\overline{I}_2$$

$$\overline{V}_1 = (-4+j6)\overline{I}_2$$

(4) $\quad \dfrac{\overline{I}_2}{\overline{I}_1} = -\dfrac{1}{4}$

(1) $240 = 2\bar{I}_1 + \bar{V}_1 = 2\bar{I}_1 + (-4+j6)\bar{I}_2 = 240$

(4) $\bar{I}_1 = -4\bar{I}_2$

(1) $-8\bar{I}_2 + (-4+j6)\bar{I}_2 = 240$

$$\boxed{\bar{I}_2 = \frac{240}{(-12+j6)}} \rightarrow \bar{V}_o = -j24\,\bar{I}_2 = \frac{-j24\times240}{-12+j6}$$

$$= \frac{-j4\times240}{-2+j} = \frac{-j960}{-2+j}$$

$$\bar{V}_o = \frac{-j960}{(-2+j)}\frac{(-2-j)}{(-2-j)} = \frac{960(2j-1)}{5}$$

$$= 192(2j-1)$$

$$= 192\sqrt{5}\,\angle 180-\tan^{-1}\left(\frac{2}{1}\right)$$

$$\bar{V}_o = 429.3\,\angle 116.57^{\circ}\ V$$

$$\bar{I}_1 = -4\bar{I}_2$$

$$= \frac{-960}{-12+j6}$$

$$\bar{S} = -240\angle 0^{\circ}\cdot \bar{I}_1^{*} = -240\cdot\frac{-960}{-12-j6} = -\frac{240\times960}{12+j6}$$

$$= -\frac{40\times960}{2+j} = -\frac{40\times960}{\sqrt{5}}\angle -\tan^{-1}(2)$$

$$= 17.17\,\angle -26.57^{\circ}\ kVA$$

$$\bar{S} = -1.5359\times10^{4} + j\,7681\ VA$$

Calculate the power supplied to the 10Ω resistor in the ideal transformer circuit of Fig. 13.39.

**Example 13.9**

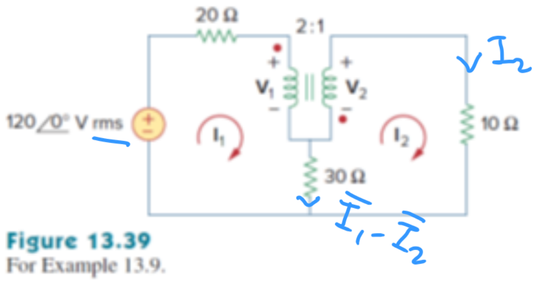



**Figure 13.39**
For Example 13.9.

$$P_{10\Omega} = |\overline{I}_2|^2 \times 10 \text{ W}$$

(1) $-120 + 20\overline{I}_1 + \overline{V}_1 + 30(\overline{I}_1 - \overline{I}_2) = 0$

(2) $-\overline{V}_2 + 10\overline{I}_2 - 30(\overline{I}_1 - \overline{I}_2) = 0$

(3) $\dfrac{\overline{V}_2}{\overline{V}_1} = -\dfrac{1}{2} \rightarrow \overline{V}_1 = -2\overline{V}_2$

(4) $\dfrac{\overline{I}_2}{\overline{I}_1} = -2 \rightarrow \overline{I}_2 = -2\overline{I}_1$

(1) $-120 + 20\overline{I}_1 - 2\overline{V}_2 + 30\overline{I}_1 + 60\overline{I}_1 = 0$

$$\boxed{120 = 110\overline{I}_1 - 2\overline{V}_2}$$

(2) $-\overline{V}_2 - 20\overline{I}_1 - 30\overline{I}_1 - 60\overline{I}_1 = 0$

$\overline{V}_2 = -110\overline{I}_1$

(1) ve (2) $120 = -\overline{V}_2 - 2\overline{V}_2 \Rightarrow \boxed{\overline{V}_2 = -40 \text{ V}_{rms}}$

$$\overline{I}_1 = \frac{\overline{V}_2}{-110} = \frac{40}{110} \text{ A}_{rms}$$

$$\overline{I}_2 = -2\overline{I}_1 = \frac{-80}{110} \text{ A}_{rm}$$

$$P_{10\Omega} = \left(\frac{80}{110}\right)^2 \times 10 = 5{,}29 \text{ W}$$

## Practice Problem 13.9

Find $\mathbf{V}_o$ in the circuit of Fig. 13.40.

$\overline{V}_o = 8\,\overline{I}_3$
?

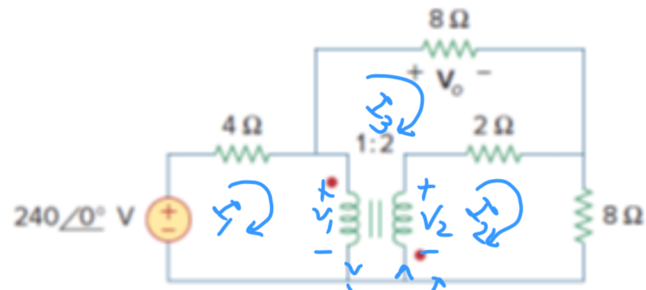



**Figure 13.40**
For Practice Prob. 13.9.

**Answer:** 96 V.

(1) $-240 + 4I_1 + V_1 = 0$

(2) $-V_2 + 2(I_2 - I_3) + 8I_2 = 0$

(3) $8I_3 + 2(I_3 - I_2) + V_2 - V_1 = 0$

(4) $\dfrac{V_2}{V_1} = -2 \Rightarrow V_2 = -2V_1$

(5) $\dfrac{I_2 - I_3}{I_1 - I_3} = -\dfrac{1}{2} \rightarrow 2I_2 - 2I_3 = -I_1 + I_3$

$\boxed{I_1 + 2I_2 = 3I_3}$

$I_3 = \dfrac{I_1 + 2I_2}{3}$

$I_1 = 3I_3 - 2I_2$

$I_1 = 3(I_3 - I_2) + I_2$

$I_3 - I_2 = \dfrac{I_1 - I_2}{3}$

(2) $-V_2 - \dfrac{2}{3}(I_1 - I_2) + 8I_2 = 0$

(3) $\dfrac{8}{3}(I_1 + 2I_2) + \dfrac{2}{3}(I_1 - I_2) - 3V_1 = 0$

$$2V_1 = \frac{2}{3} I_1 - \frac{2}{3} I_2 - 8 I_2 \quad (2)$$

$$3V_1 = \frac{10}{3} I_1 + \frac{14}{3} I_2 \quad (3)$$

$$3V_1 = I_1 - I_2 - 12 I_2 = I_1 - 13 I_2 \quad (2)$$

$$3V_1 = \frac{10 I_1}{3} + \frac{14}{3} I_2 \quad (3)$$

$$I_1 - 13 I_2 = \frac{10 I_1}{3} + \frac{14}{3} I_2$$

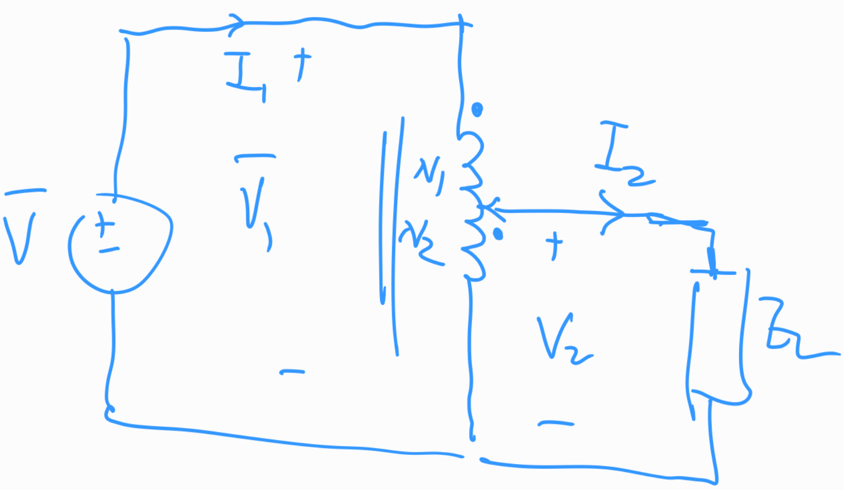
$$3 I_1 - 39 I_2 = 10 I_1 + 14 I_2$$

$$\boxed{-7 I_1 = 53 I_2}$$

Sonuç: $\bar{V}_0 = 96V$

## 13.6 ideal Ototrafo (Autotransformer)

Tek bir sarım var (Toplam $N_1 + N_2$ sarım var)




Refer to the autotransformer circuit in Fig. 13.44. Calculate: (a) $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_o$ if $\mathbf{Z}_L = 8 + j6\ \Omega$, and (b) the complex power supplied to the load.

**Example 13.11**

$S_1 = V_1 I_1^* = S_2 = V_2 I_2^*$

$\frac{V_2}{I_1} = \frac{I_1}{I_2} = \frac{5}{2}$

$\frac{V_2}{V_1} = \frac{200}{80} = \frac{5}{2}$ ✓

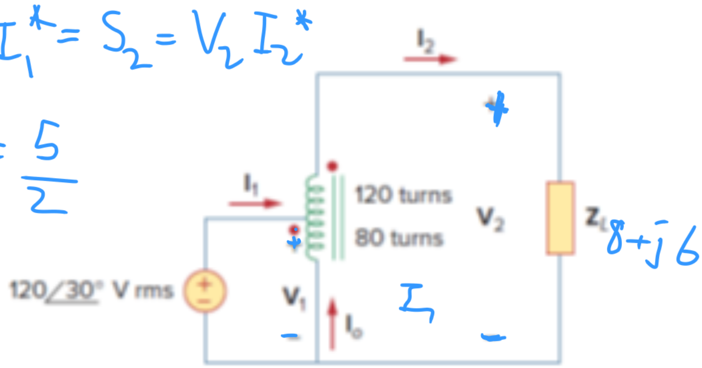



**Figure 13.44**
For Example 13.11.

$\bar{V}_1 = 120\angle 30°$ V

$\bar{V}_2 = 300\angle 30°$ V

$I_1 = \frac{5}{2} I_2 = 75\angle -7°$ A

$\bar{S} = -120\angle 30° \cdot I_1^*$

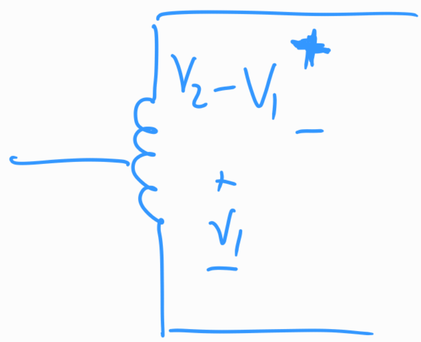
$= -120\angle 30° \cdot 75\angle 7°$

$= -9000\angle 37°$ VA

$= -7200 - j5400$ VA

$\bar{I}_2 = \frac{\bar{V}_2}{\bar{Z}_L} = \frac{300\angle 30°}{10\angle 37°}$

$\bar{I}_2 = 30\angle -7°$ A




$\frac{V_2 - V_1}{V_1} = \frac{12}{8}\ \frac{3}{2}$

$2V_2 - 2V_1 = 3V_1$

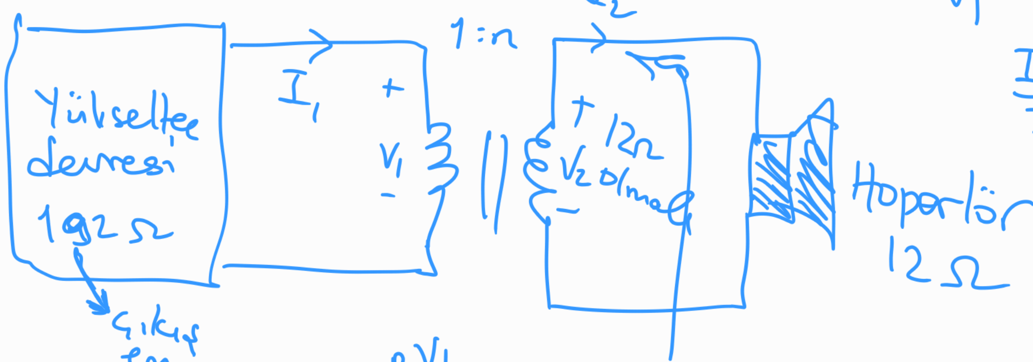
$2V_2 = 5V_1$

$\frac{V_2}{V_1} = \frac{5}{2}$

# 13.9 UYGULAMALAR

## Empedans Uyumlama

Example 13.16




çıkış empedansı

$$\frac{V_2}{V_1} = n$$

$$\frac{I_2}{I_1} = \frac{1}{n}$$

$$\frac{\overset{nV_1}{V_2}}{\underset{I_1/n}{I_2}} = 12$$

$$n^2 \underbrace{\frac{V_1}{I_1}}_{192\Omega} = 12$$

$$\cancel{192}\, n^2 = \cancel{12} \quad \frac{1}{16}$$

$$\boxed{n = \frac{1}{4}}$$

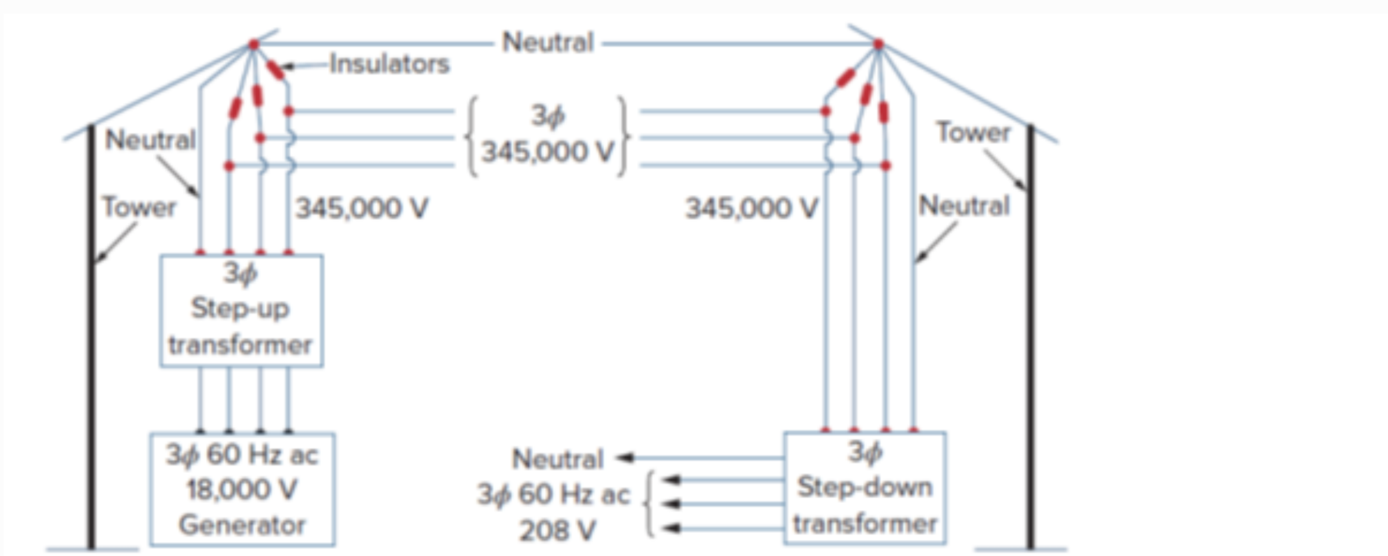



**Figure 13.67**
A typical power distribution system.