

Fair Scheduling in OFDMA-based Wireless Systems with QoS Constraints

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Abstract—In this work we consider the problem of downlink resource allocation for proportional fairness of long term received rates of data users and quality of service for real time sessions in an OFDMA-based wireless system. The base station allocates available power and bandwidth to individual users based on long term average received rates, QoS based rate constraints and channel conditions. We solve the underlying constrained optimization problem and propose an algorithm that achieves the optimal allocation. Numerical evaluation results show that the proposed algorithm provides better QoS to voice and video sessions while providing more and fair rates to data users in comparison with existing schemes.

I. INTRODUCTION

Recent IEEE 802.16 standard defines the air interface and medium access control (MAC) specifications for wireless metropolitan area networks. Such networks intend to provide high speed voice, data and on demand video streaming services for end users. IEEE 802.16 standard is often referred to as WiMax and it provides substantially higher rates than cellular networks. Besides it eliminates the costly infrastructure to deploy cables, therefore it is becoming an alternative to cabled networks, such as fiber optic and DSL systems [1],[2].

Transmissions in IEEE 802.16 networks is based on Orthogonal frequency division multiplexing (OFDM), which is a multicarrier transmission technique that is proposed for high speed wireless transmission. It is based on a large number of orthogonal subchannels, each working at a different frequency. Within OFDMA framework, the resource allocated to the users come in three dimensions: time, frequency and power. This requires the scheduler to operate with higher degree of freedom but also makes the notion of resource fairness obsolete and makes the problem more involved.

There are three main issues that need to be considered in multiple access resource allocation, which are spectral efficiency, fairness and Quality of Service (QoS). These are often contradicting aims and achieving these in an OFDMA based system is a challenging task. OFDMA-based scheduling for systems with heterogeneous traffic has not been studied sufficiently. Recently in [3], [4], [5], proportional fair scheduling was studied for multicarrier systems. However, the scheduling rules in these works do not apply sufficiently to different QoS requirements and heterogeneous traffic. Besides they only provide fairness for short term received rate, instead of long term. In this work our goal is to find algorithms that provide proportional fairness to data users and QoS for real time sessions such as voice and video streaming.

II. SYSTEM MODEL

We consider a downlink system, where a base station transmits to N users. Each user is assumed to demand a single type of traffic. Let W and P denote the total bandwidth and power, respectively. Total bandwidth W is divided into N_{sub} subchannels of length W_{sub} , each consisting of a group of carriers. We consider PUSC as the mode of subchannelization, which is more suitable for mobile users than AMC. It provides frequency diversity and inter-cell interference averaging. This minimizes the performance degradation due to fast fading characteristics of mobile environments. We assume flat fading, which is a reasonable assumption with PUSC.

The noise and interference power density is N_0 , and the channel gain averaged over the entire band from the BS to user i at time t is $h_i(t)$, where $h_i(t)$ includes path loss, shadowing (lognormal fading) and fast fading. There are three classes of users. Users in the classes U_D , U_S and U_V demand data, voice and video traffic, respectively. The system that

we consider is time slotted with frame length T_s . The scheduler makes a resource allocation decision at each frame. Active period in a voice conversation, streaming duration and file size are both very long with respect to the frame size. Therefore it is realistic that during the course of optimization the number of active sessions are fixed.

In problem formulation we will adopt the Shannon channel capacity for AWGN channel as a function for bandwidth and transmission power assigned to user i :

$$r_i(w_i(t), p_i(t)) = w_i(t) \log \left(1 + \beta \frac{p_i(t)h_i(t)}{N_0 w_i(t)} \right) \quad (1)$$

The reason for using Shannon capacity is its simplicity, and it also approximates the above set of rate-SINR relation with $\beta = 0.25$. The parameter $0 < \beta < 1$ compensates the rate gap between Shannon capacity and rate achieved by practical modulation and coding pairs listed in IEEE 802.16a/e standards. After finding power and bandwidth pairs by solving the convex programming problem, we quantize the SINRs to the thresholds determined in the standard. In this paper, we propose a Fair and QoS-based Power and Subchannel Allocation (FQPSA) Algorithm. Objective of the algorithm is to maximize proportional fairness for data users subject to rate constraints of real time users selected each frame according to a metric.

III. PROBLEM FORMULATION

A. Data Traffic

The objective for the data users is to optimize log sum of the exponential averaged rates of the users. Average received user rates are updated at each frame according to the exponential averaging formula:

$$R_i(t) = \alpha_i R_i(t-1) + (1 - \alpha_i) r_i(w_i(t), p_i(t)), \forall i, t \quad (2)$$

This way we consider both current rate as well as rates given to the user in the past. Observed at time t , the highest consideration is given to the current rate $r(t)$, and the rates received at the past $t-1, t-2, \dots$ carry diminishing importance.

$$C(\mathbf{R}(t)) = \sum_{i=1}^N \log R_i(t) = C(\mathbf{R}(t-1)) + \sum_{i=1}^N \log \left(\alpha + \frac{\bar{\alpha} r_i(w_i(t), p_i(t))}{R_i(t-1)} \right) \quad (3)$$

As a matter of fact, we limit ourself to *greedy* schemes in the sense that at frame t , we try to

maximize the proportional fair capacity $C(\mathbf{R}(t))$ without considering the future frames $t+1, t+2, \dots$. Only the second term in (3) needs consideration. Here the log-sum can be replaced with product.

B. Real Time Traffic

The Base Station first chooses a number of voice and video users according to a metric that reflects current channel condition, head of line packet delay and long term received rate. Then for the selected users the number of bits to be transmitted in the current frame is determined. These procedures are explained in [6], in detail. Let U'_R be the set of real time sessions selected and let r_i^c be the rate requirement of real time session i .

The resulting optimization problem considering all types of sessions is as follows: Find,¹:

$$(\mathbf{p}^*, \mathbf{w}^*) = \arg \max_{\mathbf{p}, \mathbf{w}} \prod_{i \in U_D} \left(\alpha_i + \frac{\bar{\alpha}_i w_i \log \left(1 + \frac{p_i}{n_i w_i} \right)}{R_i} \right)$$

subject to,

$$\sum_{i \in U_D \cup U'_R} p_i^* \leq P \quad (4)$$

$$\sum_{i \in U_D \cup U'_R} w_i^* \leq W \quad (5)$$

$$w_i^* \log \left(1 + \frac{p_i^*}{n_i w_i^*} \right) \geq r_i^c, i \in U'_R \quad (6)$$

$$p_i^*, w_i^* \geq 0, \forall i \in U_D \cup U'_R \quad (7)$$

where $n_i = \frac{N_0}{\beta h_i}$.

IV. SOLUTION OF THE OPTIMIZATION PROBLEM

The objective function (4) is an increasing function of (w, p) , therefore the maximum is achieved only when the constraints (4, 5, 6) are all met with equality. For this reason we can replace these inequalities with equalities in the discussion below.

Actually there is no guarantee that a solution can be found that satisfies both rate and power constraints, given the rate requirements and channel conditions. The rate requirement can be too high that it may be impossible to satisfy with the given channel conditions. To start with, we assume that the problem is *feasible*. We will discuss about how to detect infeasibility of the problem and what to do in that case in the next section. We can write the Lagrangian of the problem as:

¹Here p_i, w_i, n_i are the values at time t . The time index is not shown for convenience.

$$L(\mathbf{w}, \mathbf{p}, \lambda_p, \lambda_w, \lambda^r) = \prod_{i \in U_D} \left(\alpha_i + \frac{\bar{\alpha}_i w_i \log \left(1 + \frac{p_i}{n_i w_i} \right)}{R_i} \right) + \lambda_p \left(P - \sum_{i \in U_D \cup U'_R} p_i \right) + \lambda_w \left(W - \sum_{i \in U_D \cup U'_R} w_i \right) + \sum_{i \in U'_R} \lambda_i^r \left(w_i \log \left(1 + \frac{p_i}{n_i w_i} \right) - r_i^0 \right). \quad (8)$$

From now on we assume that all rates are in nats/sec and logarithms are natural. Taking the derivatives of $L(\mathbf{p}, \mathbf{w}, \lambda_p, \lambda_w, \lambda^r)$, we get the following:

A. For users $i \in U_D$

$$\frac{\partial L(\mathbf{p}, \mathbf{w}, \lambda_p, \lambda_w, \lambda^r)}{\partial p_i} \Big|_{(\mathbf{p}^*, \mathbf{w}^*)} = 0 \Rightarrow \frac{L^*}{\lambda_p} = (n_i w_i^* + p_i^*) \left(\frac{R_i \tilde{\alpha}_i}{w_i^*} + \log \left(1 + \frac{p_i^*}{n_i w_i^*} \right) \right) \quad (9)$$

$$\frac{\partial L(\mathbf{p}, \mathbf{w}, \lambda_p, \lambda_w, \lambda^r)}{\partial w_i} \Big|_{(\mathbf{p}^*, \mathbf{w}^*)} = 0 \Rightarrow \frac{L^*}{\lambda_w} = \frac{(n_i w_i^* + p_i^*) \left(R_i \tilde{\alpha}_i + w_i^* \log \left(1 + \frac{p_i^*}{n_i w_i^*} \right) \right)}{(n_i w_i^* + p_i^*) \log \left(1 + \frac{p_i^*}{n_i w_i^*} \right) - p_i} \quad (10)$$

where $\tilde{\alpha}_i = \frac{\alpha_i}{1 - \alpha_i}$. Let $\Lambda_p = \frac{L^*}{\lambda_p}$ and $\Lambda_w = \frac{L^*}{\lambda_w}$. By dividing (9) with (10) we can write for all $i \in U_D$:

$$\frac{\Lambda_p}{\Lambda_w} = \Lambda_a = n_i ((1 + x_i^*) \log (1 + x_i^*) - x_i^*), \quad (11)$$

where $x_i^* = \frac{p_i^*}{n_i w_i^*}$ denotes the optimal *effective* SINR, which is the SINR multiplied by the SINR gap parameter β .

B. For users $i \in U'_R$

$$\frac{\partial L(\mathbf{p}, \mathbf{w}, \lambda_p, \lambda_w, \lambda^r)}{\partial p_i} \Big|_{(\mathbf{p}^*, \mathbf{w}^*)} \Rightarrow \frac{\lambda_p}{\lambda_i^v} = \frac{1}{n_i} \frac{1}{1 + \frac{p_i^*}{n_i w_i^*}} \quad (12)$$

$$\frac{\partial L(\mathbf{p}, \mathbf{w}, \lambda_p, \lambda_w, \lambda^r)}{\partial w_i} \Big|_{(\mathbf{p}^*, \mathbf{w}^*)} \Rightarrow \frac{\lambda_w}{\lambda_i^r} = \log \left(1 + \frac{p_i^*}{n_i w_i^*} \right) - \frac{\frac{p_i^*}{n_i w_i^*}}{1 + \frac{p_i^*}{n_i w_i^*}} \quad (13)$$

With $\frac{\Lambda_p}{\Lambda_w} = \frac{\lambda_w}{\lambda_p}$, dividing equation (13) to (12) for all $i \in U'_R$ again gives (11).

By dividing (13) to (12) we can eliminate λ_i^v 's from the problem. It is worth noting that we get the same relation between Λ_a/n_i and x_i for both data and real time sessions. At this point it is convenient to define the function $f_a(x)$ as:

$$f_a(x) = (1 + x) \log(1 + x) - x. \quad (14)$$

Then we have,

$$x_i = f_a^{-1}(\Lambda_a/n_i), \forall i \in U_D \cup U'_R \quad (15)$$

Signal to noise ratio (x_i) is a monotonic increasing function of Λ_a for all users $i \in U_D \cup U'_R$.

For real time users we also have:

$$\frac{\partial L(\mathbf{p}, \mathbf{w}, \lambda_p, \lambda_w, \lambda^r)}{\partial \lambda_i^v} \Big|_{(\mathbf{p}^*, \mathbf{w}^*)} = 0 \Rightarrow r_i^0 = w_i^* \log \left(1 + \frac{p_i^*}{n_i w_i^*} \right), \forall i \in U'_R \quad (16)$$

C. For all nodes $i \in U_D \cup U'_R$

$$\frac{\partial L(\mathbf{p}, \mathbf{w}, \lambda_p, \lambda_w, \lambda^r)}{\partial \lambda_p} \Big|_{(\mathbf{p}^*, \mathbf{w}^*)} = 0 \Rightarrow P = \sum_{i \in U} p_i^* \quad (17)$$

$$\frac{\partial L(\mathbf{p}, \mathbf{w}, \lambda_p, \lambda_w, \lambda^r)}{\partial \lambda_w} \Big|_{(\mathbf{p}^*, \mathbf{w}^*)} = 0 \Rightarrow W = \sum_{i \in U} w_i^* \quad (18)$$

From Equation (9) for data users we can write:

$$\frac{[\Lambda_p - n_i(1 + x_i^*) R_i \tilde{\alpha}_i]^+}{\log(1 + x_i^*)(1 + x_i^*) n_i} = w_i^*, i \in U_D \quad (19)$$

$$\frac{[\Lambda_p - n_i(1 + x_i^*) R_i \tilde{\alpha}_i]^+ x_i^*}{\log(1 + x_i^*)(1 + x_i^*)} = p_i^*, i \in U_D \quad (20)$$

The $[.]^+$ operator in Equations (19),(20) guarantees that $w_i, p_i \geq 0$ for all users. Given Λ_p and Λ_a we can compute the power and bandwidth for users $i \in U_D$ using (19) and (20). Given Λ_a , we can calculate the power and bandwidth for users $i \in U'_R$ using (16). Please note that just from (16), (19) and (20), the bandwidth and power constraints (5) (4) are *not* necessarily satisfied. We need to find the right Λ_a and Λ_p so that the power and bandwidth constraints are satisfied. Let $S_p(\Lambda_a, \Lambda_p)$ and $S_w(\Lambda_a, \Lambda_p)$ be the total bandwidth and total power corresponding to Λ_a and Λ_p :

$$S_w(\Lambda_a, \Lambda_p) = \sum_{i \in U_D \cup U'_R} w_i(\Lambda_a, \Lambda_p) = \sum_{i \in U_D} \frac{[\Lambda_p - n_i(1 + x_i) R_i \tilde{\alpha}_i]^+}{\log(1 + x_i)(1 + x_i) n_i} + \sum_{i \in U'_R} \frac{r_i^c}{\log(1 + x_i)} \quad (21)$$

$$\begin{aligned}
S_p(\Lambda_a, \Lambda_p) &= \sum_{i \in U_D \cup U'_R} p_i(\Lambda_a, \Lambda_p) = \\
&\sum_{i \in U_D} \frac{[\Lambda_p - n_i(1+x_i)R_i \tilde{\alpha}_i]^+ x_i}{\log(1+x_i)(1+x_i)} + \sum_{i \in U'_R} \frac{r_i^c x_i n_i}{\log(1+x_i)}
\end{aligned} \tag{22}$$

where $x_i = f_a^{-1}(\Lambda_a/n_i)$ is the SNR of user i . As a result, the problem is finding Λ_a^* and Λ_p^* such that

$$S_w(\Lambda_a^*, \Lambda_p^*) = W \tag{23}$$

$$S_p(\Lambda_a^*, \Lambda_p^*) = P \tag{24}$$

using Equations (15), (23) and (24). Note that although Λ_a and Λ_p are independent variables that determine power and bandwidth for each node, they become dependent when the power and bandwidth constraints (23) (24) need to be satisfied.

D. Feasibility of the Solution

From (19) and (20), $\Lambda_p = 0$ corresponds to the case that no bandwidth and power is allocated to data sessions. If the problem is feasible, then there exists a Λ_a that satisfies the following:

$$S_w(\Lambda_a, 0) = W, \tag{25}$$

$$S_p(\Lambda_a, 0) \leq P. \tag{26}$$

Let Λ_a^0 be the smallest Λ_a that satisfies (25) and (26). In the report [6] we proved the existence and uniqueness of Λ_a^0 for a feasible problem using monotonicity properties of the functions $S_w(\Lambda_a, \Lambda_p)$ and $S_p(\Lambda_a, \Lambda_p)$.

V. PROPOSED ALGORITHM

Using (15), (21) and (22) we can develop an algorithm to determine the power and bandwidth allocation. The algorithm is also able to detect infeasibility if there no solution exists.

Algorithm:

- 1) Find Λ_a^0 . If $S_p(\Lambda_a^0, 0) < P$ then the problem is feasible. If not, find the real time session that consumes the largest power and decrease its rate. Repeat Step 1 and 2 until feasibility is achieved.
- 2) Using two nested binary searches find the unique $(\Lambda_a^*, \Lambda_p^*(\Lambda_a^*))$ that satisfies power and bandwidth constraints.
- 3) Compute optimal power and bandwidth using the optimal $(\Lambda_a^*, \Lambda_p^*(\Lambda_a^*))$.

Let us summarize the nested binary search. We start from a Λ_a , then we find the $\Lambda_p^*(\Lambda_a)$ that satisfies the bandwidth constraint. If the power constraint

is exceeded then we decrease Λ_a' , else increase it, and search for $\Lambda_p^*(\Lambda_a')$. We continue until both constraints are satisfied.

Since the problem is convex (proved in [6]), the problem has a unique solution. We need the monotonicity property of function $S_p(\Lambda_a, \Lambda_p^*(\Lambda_a))$ (sum of powers) as a function of Λ_a in order to be able to find it using binary search. Those properties are also verified in [6].

VI. PERFORMANCE EVALUATION

For the numerical evaluations we equally divide the users to 5 classes according to the distances, 0.3, 0.6, 0.9, 1.2, 1.5 km. We use the parameters in Table I.

Parameter	Value
Cell radius	1.5km
User Distances	0.3, 0.6, 0.9, 1.2, 1.5 km
Total power (P)	20 W
Total bandwidth (W)	8.3 MHz
Frame Length	1 msec
Voice Traffic	CBR 32kbps
Video Traffic	802.16 - 128kbps
FTP File	5 MB
AWGN p.s.d.(N_0)	-174dBm/Hz
Pathloss exponent (γ)	3.5
$\Psi_{DB} \sim N(\mu_{\Psi_{DB}}, \sigma_{\Psi_{DB}})$	$N(0dB, 8dB)$
Coherent Time (Fast/Slow Fading)	(5msec/300msec.)
Pathloss(dB, d in meters)	$-31.5 - 35 \log_{10} d + \Psi_{dB}$

TABLE I
SIMULATION PARAMETERS

We will use M-LWDF-PF scheme as benchmark in our simulations. In this scheme at each frame the user maximizing the following quantity transmits.

$$a_i D_i^{HOL}(t) \log(1 + \beta P/N_0 W), \tag{27}$$

where $D_i^{HOL}(t)$ is the head of line packet delay and $r_i(t)$ is the channel capacity of user i at frame t . The parameter a_i is a positive constant. The constant a_i is defined as $a_i = -\frac{\log(\delta_i)}{D_i^{max} R_i(t)}$, which is referred to as M-LWDF-PF [7] [4]. M-LWDF-PF can be adapted to OFDMA systems as follows. Power is distributed equally to all subchannels. Starting from the first subchannel, the subchannel is allocated to the user maximizing (27). Then the received rate $R(t)$ is updated according to (2). All the subchannels are allocated one-by-one according to this rule. Delay exceeding probability is taken as $\delta_i = 0.05$ for all users. Delay constraint for voice, video and data users are 0.1, 0.4 and 1 seconds, respectively.

Tables II, III and IV show the effects of increasing the number of voice, video streaming and data

users, respectively. In these tables we observe that FPQS Algorithm is better than M-LWDF algorithm in terms of both delay performance and data performance. With FQPS delay for voice and video sessions stay within acceptable bounds, while with M-LWDF, it exceeds the bounds for the user at the cell edge when $V \geq 30$ or $S > 40$. Besides, FQPSA provides at least 10 percent increase in total throughput. Total throughput decreases linearly with increasing number of voice and video users. Although 10 voice users adds up to 0.32 Mbps, adding 10 users decreases the total throughput approximately by 1.2-1.4 Mbps. This is because voice has a very strict delay requirement and a voice session may have to be transmitted despite bad channel conditions. Throughput for LWDF decreases with a little bit slower rate but that reflects to the voice and video performance negatively. Log-sum performance of FQPSA is also better than that of M-LWDF, which shows that our algorithm provides fairness.

V =	10	20	30	40	50	60
FQPS Voice (G)	36	33	30	32	31	31
FQPS Voice (B)	88	66	62	81	82	80
LWDF Voice (G)	41	41	41	41	41	41
LWDF Voice (B)	89	91	93	110	133	151
FQPS Video (G)	148	132	130	132	132	131
FQPS Video (B)	295	271	250	263	266	252
LWDF Video (G)	180	187	192	200	202	208
LWDF Video (B)	384	399	408	410	423	435
FQPS (Mbps)	26.9	25.2	23.8	22.7	21.4	20.1
FQPS log-sum	280	279	278	277	276	275
LWDF (Mbps)	23.9	22.7	21.6	20.5	19.4	18.5
LWDF Log-sum	275	278	276	274	272	269

TABLE II

EFFECTS OF INCREASING # OF VOICE USERS FOR $S = 20, D = 20$. UNITS: DELAY (MSEC), T.PUT (MBPS)

In Table IV we can observe the effects of increasing the number of data users. We observe that delay for both voice and video streaming sessions stay approximately constant. Delay performance is much better than that of M-LWDF algorithm. Data performance is 10 percent better than M-LWDF. Total throughput increases with number of data users, but the increase diminishes as D increases.

REFERENCES

S =	10	20	30	40	50	60
FQPS Voice (G)	31	33	39	43	49	51
FQPS Voice (B)	57	66	71	87	93	108
LWDF Voice (G)	41	41	41	41	41	41
LWDF Voice (B)	86	91	94	101	142	117
FQPS Video (G)	122	132	150	167	174	190
FQPS Video (B)	217	271	275	301	338	392
LWDF Video (G)	171	187	190	194	202	223
LWDF Video (B)	395	399	399	414	463	519
FQPS D(Mbps)	27.2	25.2	23.5	21.5	19.5	17.5
FQPS Log-sum	281	279	278	276	274	272
LWDF (Mbps)	25.1	22.7	20.7	18.6	16.7	14.6
LWDF log-sum	279	277	277	276	275	273

TABLE III
EFFECTS OF INCREASING # OF VIDEO STREAMING USERS FOR $D = 20, V = 20$. UNITS: DELAY (MSEC), T.PUT (MBPS)

D =	20	30	40	50	60	70
FQPS Voice (G)	38	33	34	34	36	39
FQPS Voice (B)	72	66	69	71	71	91
LWDF Voice (G)	41	47	50	57	61	67
LWDF Voice (B)	91	106	126	146	170	182
FQPS Video (G)	155	132	143	145	142	168
FQPS Video (B)	278	271	246	270	270	323
LWDF Video (G)	187	232	265	297	335	357
LWDF Video (B)	399	460	537	557	612	656
FQPS (Mbps)	24.5	25.2	25.8	26.1	26.2	26.6
FQPS Log-sum	146	279	407	532	654	775
LWDF (Mbps)	22.7	23.3	23.7	24.0	24.1	24.3
LWDF Log-sum	146	278	405	530	653	772

TABLE IV
EFFECTS OF INCREASING # OF DATA USERS FOR $S = 20, V = 20$. UNITS: DELAY (MSEC), T.PUT (MBPS)

[1] C. Eklund, R. B. Marks, K.L. Stanwood, and S. Wang. IEEE Standard 802.16: A Technical Overview of the WirelessMAN Air Interface for Broadband Wireless Access. *IEEE Communications Magazine*, June 2002.

[2] A. Ghosh, D. Wolter, J. G. Andres, and R. Chen. Broadband Wireless Access with WiMax/802.16: Current Performance Benchmarks and Future Potential. *IEEE Communications Magazine*, Feb. 2005.

[3] H. Kim and Y. Han. A Proportional Fair Scheduling for Multicarrier Transmission Systems. *IEEE Communication Letters*, pages 210–212, Mar. 2005.

[4] G. Song and G. Li. Utility-Based Resource Allocation and Scheduling in OFDM-Based Wireless Broadband Networks. *IEEE Communications Magazine*, Dec. 2005.

[5] C. Zhu and J. Agre. Proportional-Fair Scheduling Algorithms for OFDMA-based Wireless Systems. *Preprint, Fujitsu Labs*, 2006.

[6] T. Girici, C. Zhu, J. Agre, and A. Ephremides. Proportional Fair Scheduling Algorithm in OFDMA-based Wireless Systems with QoS Constraints. *Preprint, Fujitsu Labs*, 2006.

[7] M. Andrews, K. Kumaran, K. Ramanan, A. Stolyar, and P. Whiting. Providing Quality of Service over a Shared Wireless Link. *IEEE Communications Magazine*, pages 150–154, Feb. 2001.