

# Optimizing The Service Policy Of A Wireless Access Point On The Move With Renewable Energy

Elif Tugce Ceran, Tugce Erkilic, Elif Uysal-Biyikoglu, Tolga Girici and Kemal Leblebiciooglu  
E-mail: eceran@metu.edu.tr, terkilic@aselsan.com.tr, uelif@metu.edu.tr, tgirici@etu.edu.tr, kleb@metu.edu.tr

**Abstract**—Inspired by recent industry efforts toward providing Internet access to areas of the world devoid of regular telecommunications infrastructure, an online resource allocation problem for a mobile access point (AP) is studied. While prudently managing its available energy, the AP allocates its resources to maximize the total utility (reward) provided to the users demanding service. The problem is formulated as a 0/1 dynamic knapsack problem with incremental capacity in a finite time horizon, the solution of which is quite open in the literature. The problem is approached from through stochastic and deterministic formulations. For the stochastic case, using a dynamic programming setup, the optimality of a threshold based solution is exhibited, and a simple threshold based policy which performs closely to optimal is obtained via the expected threshold method. For the deterministic formulation, several online heuristics based on an instantaneous threshold that can adapt to short-time-scale dynamics are proposed, including one with an optimal competitive ratio under a certain condition. The performance of all heuristics are comparatively studied.

## I. INTRODUCTION

This paper considers a case of exploiting renewable energy to power a mobile Internet router. We define an Access Point on the Move (APOM) as a flying platform (e.g., situated in the lower stratosphere) powered by solar energy providing service to users in different locations on its path. The APOM resource allocation problem considered in this paper has been motivated by recent industrial efforts toward providing ubiquitous Internet access by launching mobile Internet service providers (ISP) in the Earth's atmosphere [1].

We concern ourselves with the following resource allocation problem: As the APOM moves over a certain area and observes various user demands, and given an energy budget that is occasionally but arbitrarily replenished, which demands should it serve to maximize an overall utility.

### A. Related Work

Mobile service providers have certain advantages over fixed ones [2]. Various studies exist regarding mobile sinks in conventional networks that do not exploit renewable energy [3], [2]. Some of these studies focused on determining optimal paths in order to prolong network lifetime [4], [5].

In recent years, employing energy harvesting (via ambient energy sources such as solar irradiation [6], and vibrations [7]) to power transmitters of network devices such as APs has been drawing great attention from the research community. The consideration of mobile access points with energy harvest capability is relatively new. Several of the recent studies on the topic are concerned with finding optimal routing paths

[8]. Xie et al. [9] address the problem of colocating the mobile service provider on the wireless charging machine with the objective of minimizing energy consumption. The closely related works of Ren and Liang [10] considered a distributed time allocation method to maximize data collection in energy harvesting sensor networks while defining a scenario of a constrained path with all sensors having renewable energy sources.

Kashef et. al. [11] consider binary decision problem to transmit or defer several tasks according to a stochastic model based on Gilbert-Elliott channel and prove that a threshold based approach is optimal. Similar works as [12], [13], also consider threshold based solutions for the resource allocation problem in energy harvesting systems for stochastic models. In recent work [14], [15] resource allocation at solar powered stationary and mobile service providers has been addressed through various different optimization techniques.

The problem at hand can be set up as a 0/1 dynamic online Knapsack problem. While the Knapsack problem is a well known combinatorial optimization problem [16], competitive online solutions are limited. Chakrabarty et. al. proposes a constant competitive solution to the problem with static and large capacity [17]. The dynamic capacity case, which applies to the setup in this paper, is largely open.

### B. Our Contribution

This paper studies a mobile access point powered through (solar) energy harvesting, which aims to maximize the total data service it provides to users appearing to it in a sequential manner. One difference of the approach from related studies is that the problem is cast as an online user admission problem under deterministic as well as stochastic models for the user (demand) population.

The resource allocation problem is mapped to a multi constraint 0/1 knapsack problem (KP). After exhibiting the existence of a threshold based optimal policy, scalable and computationally cost effective heuristics are proposed. The performance of these heuristics are studied numerically (through simulations) and competitive ratio analyses are conducted.

In the resource allocation literature, there are a number of studies implementing the optimization tools proposed here such as genetic algorithms and rule-based logic, but to our knowledge this is the first application of these techniques to a threshold based user selection problem. Furthermore, our schemes constitute a fairly competitive solution to the

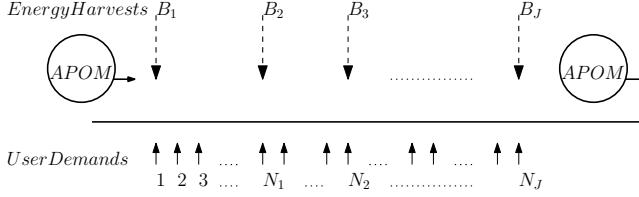


Fig. 1. System Model of APOM

dynamic knapsack problem with incremental capacity, optimal competitive ratios for which are currently not available.

### C. Organization of the paper

The system model and the setup of the optimal decision problem of which users to serve is defined as a generalized version of a *0/1 knapsack problem* in Section II. Next, the stochastic version of the problem is examined in detail and several approaches providing both optimal and suboptimal solutions are proposed in Section III. After exhibiting an optimal algorithm and the existence of a threshold for the stochastic model, the problem is investigated for a deterministic model where several novel optimization tools are employed to propose threshold based heuristic solutions to the resource allocation problem in Section IV. Detailed numerical and simulation results are presented in Section V. The paper is concluded with an outline of future directions in Section VI.

## II. SYSTEM MODEL AND PROBLEM STATEMENT

Consider an APOM (mobile Internet access point or router) relying on renewable energy resources (e.g. solar, wind) while it moves on a predefined path. As it travels on its route, decisions are to be made on an event-based schedule where each user request represents the start of a slot. We consider a finite horizon problem with  $N$  slots and  $N$  (distinct) corresponding users.

The sequential user arrival model has been motivated by the following. Operating on remote areas, it is envisioned that users with transceivers that can communicate with the APOM are distributed over a large geographical area. Thus, by the time a new user appears, the service of the previous one is complete. As every energy replenishment will constitute a change the service capacity of APOM, the problem will be examined in  $J$  subintervals regarding  $J$  harvest instances as depicted in Figure 1. Each user will be represented by a *value* and *weight* pair:  $(v_n, w_n)$  for the  $n^{th}$  user, independently from any specific channel presumption. The *value* of a user corresponds to the utility gained by serving a user whereas the *weight* corresponds to the power consumption required to serve it. APOM makes a binary decision at each slot, whether to serve the encountered user or not. Once a decision on a user has been made, there will be no re-evaluation on the same user. In the case of interest where energy replenishment rate cannot meet the power needed to serve all users, APOM has to (in an online fashion) has to pick a proper subset of users to maximize total utility under energy constraints.

In the following sections online heuristics and policies are proposed over system models constructed upon both deterministic and stochastic threshold based schemes considering user characteristics and energy harvests. For the deterministic problem formulation, following a widely adopted assumption about energy replenishment, the amount of harvested energy in certain time periods are presumed to be non-deterministic but predictable [6]. However, over a stochastic problem formulation, energy scavenging is modelled as an IID random process using the similar assumptions in [11], [14].

Both deterministic and stochastic approaches for this system setup bring discrete power levels and utilities, which is also consistent with practical concerns. Following knapsack terminology, APOM is characterized by its *capacity* to serve, which corresponds to the amount of energy stored in its battery. The main aim is to collect the maximum value over  $N$  users while ensuring that total weight does not exceed the service capacity. Stated this way, the problem is an online knapsack problem. However, in accordance with energy harvesting, we allow capacity replenishment that results in an extension of knapsack problem with dynamic incremental capacity.

### Problem 1. Service policy optimization: deterministic model

$$\text{Maximize: } \sum_{n=1}^N v_n x_n \quad (1)$$

$$\text{subject to: } \sum_{n=1}^{N_1} w_n x_n \leq B_1, \quad (2)$$

$$\sum_{n=1}^{N_2} w_n x_n \leq B_1 + B_2, \dots, \sum_{n=1}^{N_J} w_n x_n \leq \sum_{j=1}^J B_j \quad (3)$$

$$x_n \in \{0, 1\} \quad (4)$$

There is no presumed probabilistic model on user arrivals or types in deterministic setup in Problem 1, that is each users is associated with  $(v_n, w_n)$  pair. On the other hand, considering the stochastic model, demand and utility of encountered users are modelled as discrete random variables. Thus the maximization is calculated over an expectation of total value till the end of finite horizon  $N$ . There are assumed to be  $K$  types of users appearing with probability  $p(k)$  such that  $\sum_{k=1}^K p(k) = 1$  in this model. Each is associated with a utility  $(v_n(k))$  and energy demand  $(w_n(k))$  appearing at each slot  $n$ . Then, the objective of the APOM is to achieve a maximum expected utility under energy causality constraints as given in Problem 2.

**Problem 2.** *Service policy optimization: stochastic model*

$$\text{Maximize: } E \left( \sum_{n=1}^{N_J} v_n x_n \right) \quad (5)$$

$$\text{subject to: } \sum_{n=1}^{N_1} w_n x_n \leq B_1,$$

$$\sum_{n=1}^{N_2} w_n x_n \leq B_1 + B_2, \dots, \sum_{n=1}^{N_J} w_n x_n \leq \sum_{j=1}^J B_j \quad (6)$$

$$x_n \in \{0, 1\} \quad (7)$$

At the beginning of the problem horizon, there is a certain amount of energy stored in the battery. Energy is replenished (with arbitrary amounts) right after slots  $n = N_1, N_2, \dots, N_J$ . The problem is stated in terms of the decision variables  $\{x_n \in 0, 1, n = 1, 2, \dots, N\}$ 's, which indicate the decision to either serve the  $n^{th}$  user ( $x_n = 1$ ) or pass it up ( $x_n = 0$ ).

### III. OPTIMIZING THE SERVICE POLICY OF APOM THROUGH A STOCHASTIC KNAPSACK PROBLEM FORMULATION

#### A. Optimal Online Policy via Dynamic Programming

For the following analysis, a state for the  $n^{th}$  user is to be defined in terms of the available energy  $e = [1, \dots, E]$  and user type  $k$ . Let the action taken for  $n^{th}$  user be  $x_n = \{0, 1\}$  where  $x_n = 1$  means "transmit" and  $x_n = 0$  means "defer". Action space over  $N$  slots is  $\{0, 1\}^N$ . Energy arrivals  $\{Q_n \in \{0, 1\}\}$  are modelled as an IID random sequence indexed by  $n$ . In each slot, the probability of a user of type  $k$ ,  $k \in \{1, \dots, K\}$  is  $1/K$ , independently of all other slots, the value of a user of type  $k$  being  $v(k)$  and the costs are unity,  $w(k) = 1$ , without loss of all generality.

$V(e, k, n)$  denotes the expected total value from slot  $n$  starting with energy level  $e$  and user type  $k$  till the end of time horizon  $N$ , that is,  $V(e, k, n) = E(\sum_{i=n}^N V_i x_i)$ . A Dynamic Programming (DP) equation can be written to maximize this value, starting with:

$$V^*(e, k, N) = v(k), \forall e \geq 1 \quad (8)$$

For a current user  $n$ , the expectation of the total value (till the end of the horizon), after choosing to transmit to the current user, is denoted as  $V_1(e, k, n)$  whereas that after choosing denying that user is represented as  $V_0(e, k, n)$ . Comparing these quantities, the optimal expected value may be stated as:

$$V^*(e, k, n) = \max_{x_n} V_{x_n}(e, k, n) = \max\{V_1(e, k, n), V_0(e, k, n)\} \quad (9)$$

Backward induction of DP reveals a threshold based policy where APOM adopts a conservative attitude at initial slots and turns into a Greedy form towards the end of the horizon,  $N$ . APOM attempts to serve users with higher utility as long as the energy constraints are satisfied, which implies that the residual energy of the APOM should be greater than the weight of the

corresponding user. A pseudo code of the optimal solution is given in Algorithm 1.

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#### Algorithm 1 DP solution to the problem for finite horizon

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for  $e = 0$  to  $E$  do
  for  $k = 1$  to  $K$  do
     $V(e, k, N + 1) = 0$  {Initialization step}
  end for
end for
for  $n = N$  to  $1$  do
  for  $e = 0$  to  $E$  do
    for  $k = 1$  to  $K$  do
      if  $w(k) > e$  then
         $V(e, k, n) = E_{(k', Q)}\{V^*(e + Q, k', n + 1)\}$ 
      else
         $V(e, k, n) = \max\{\mathbb{E}_{(k', Q)}\{V^*(e + Q, k', n + 1)\}, v(k) + \mathbb{E}_{(k', Q)}\{V^*(e - w(k) + Q, k', n + 1)\}\}$  {Recursive equation}
      end if
      return  $V(e, k, n)$ 
    end for
  end for
end for

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#### B. Structure of the Optimal Policy

The structure of the optimal policy may be obtained based on the DP relaxation.

##### 1) Existence of threshold:

**Lemma 1.** *For a given  $k$  and  $n$ , the state defined as expected total reward  $V_x(e, k, n)$  is super-modular in available energy and decision pair  $(e, x)$ , that is, for any  $0 \leq e_0 < e_1 < \infty$ ,  $V_1(e_1, k, n) + V_0(e_0, k, n) \geq V_0(e_1, k, n) + V_1(e_0, k, n)$  for  $1 \leq n \leq N$ .*

*Proof.* The stated super-modularity corresponds the statement:

$$V_1(e_1, k, n) - V_0(e_1, k, n) \geq V_1(e_0, k, n) - V_0(e_0, k, n) \quad (10)$$

In our construction energy harvests are in increments of 1 hence there is an integer difference between  $e_1$  and  $e_0$ . It suffices to prove this statement for  $e_1 - e_0 = 1$  (it can be easily extended to higher differences by iteration of the same argument.) Let  $e_1 = e_0 + 1 = e + 1$ . The equality above can be shown to hold for  $1 \leq n \leq N$  by the following argument:

$$\begin{aligned}
 V_1(e, k, n) - V_0(e, k, n) &= v(k) \\
 &+ \frac{1}{K} \sum_{k'=1}^K q(V(e, k', n + 1) - V(e + 1, k', n + 1)) \\
 &+ (1 - q)(V(e - 1, k', n + 1) - V(e, k', n + 1)) \quad (11)
 \end{aligned}$$

and,

$$\begin{aligned}
 V_1(e + 1, k, n) - V_0(e + 1, k, n) &= v(k) \\
 &+ \frac{1}{K} \sum_{k'=1}^K q(V(e + 1, k', n + 1) - V(e + 2, k', n + 1)) \\
 &+ (1 - q)(V(e, k', n + 1) - V(e + 1, k', n + 1)) \quad (12)
 \end{aligned}$$

By subtracting (11) from (12), a sufficient condition for (10)

to hold  $\forall n \geq 1$  becomes:

$$V(e, k, n) - V(e - 1, k, n) \geq V(e + 1, k, n) - V(e, k, n) \quad (13)$$

Then, the condition of (13) is proved by induction. First, the condition is satisfied when  $n = 1$ , that is both sides of the equation are equal to 0. Second, if it is true for some  $n - 1$  then it also holds for  $n$ .

$$V(e, k, n) - V(e - 1, k, n) \geq V(e + 1, k, n) - V(e, k, n) \quad (14)$$

We will examine the three cases corresponding 3 energy states  $(e + 1, e, e - 1)$  and the three decisions  $(d_1, d_2, d_3 \in \{0, 1\})$  respectively.

$$\begin{aligned} & V_{d1}(e + 1, k, n) \\ & -V_{d2}(e, k, n) - (V_{d2}(e, k, n) - V_{d3}(e - 1, k, n)) \leq 0 \end{aligned} \quad (15)$$

$$\begin{aligned} & V_{d1}(e + 1, k, n) - V_{d1}(e, k, n) + V_{d1}(e, k, n) - V_{d2}(e, k, n) \\ & - (V_{d2}(e, k, n) - V_{d3}(e - 1, k, n)) \\ & - V_{d3}(e, k, n) + V_{d3}(e, k, n) \leq 0 \end{aligned} \quad (16)$$

By optimality of  $d_2$  for energy state  $e$ ,  $V_{d1}(e, k, n) - V_{d2}(e, k, n)$  statement is already smaller and equal to 0. Same property holds for the  $V_{d3}(e, k, n) - V_{d2}(e, k, n)$  statement. Therefore, we should only consider the remaining terms. For each possible case of  $[d_1, d_3] \in \{0, 1\}^2$ , the inequality in (16) is shown to be satisfied. For example, lets examine the case where  $d_1 = 1, d_2 = 1$ :

$$\begin{aligned} & V_1(e + 1, k, n) - V_1(e, k, n) - (V_1(e, k, n) - V_1(e - 1, k, n)) \\ & = \sum_{k'=1}^K p(k)q(V(e + 1, k, n - 1) - V(e, k, n - 1) \\ & \quad - V(e, k, n - 1) + V(e - 1, k, n - 1)) \\ & \quad + (1 - q)(V(e, k, n - 1) - V(e - 1, k, n - 1) \\ & \quad - V(e - 1, k, n - 1) + V(e - 2, k, n - 1)) \leq 0 \end{aligned} \quad (17)$$

The above inequality holds since the difference is assumed to be non increasing in available energy ( $e$ ). Similar steps may be followed for all combinations of  $d_1$  and  $d_3$  where  $[d_1, d_3] \in \{0, 1\}^2$ . Hence, the total expected reward is a supermodular function in  $(d, e)$ .  $\square$

**Theorem 1.** *The optimal policy is a threshold type policy in the available energy at each slot  $n$  and there is a threshold  $\eta$  defined as:  $x_n(k) = \begin{cases} 1 & : e \geq \eta(n, k) \\ 0 & : e < \eta(n, k) \end{cases}$*

*Proof.* Let  $\{e_1, e_2, e_3\}$  be the available energies at three decision instants such that  $e_1 < e_2 < e_3$ . Suppose there exists an optimal policy which chooses to transmit at energy levels  $e_1$  and  $e_3$  while denying the user at the energy level  $e_2$ . This contradicts Lemma 1. Therefore, the crossover from "Defer" to "Transmit" happens only once as  $e$  is increased (holding all

other parameters constant), i.e. there is a threshold.  $\square$

## 2) Monotonicity of threshold:

**Lemma 2.** *Expected total reward  $V_x(e, k, n)$  is super-modular in slot index and decision pair  $(n, x)$ , that is  $V_1(e, k, n + 1) + V_0(e, k, n) \geq V_0(e, k, n + 1) + V_1(e, k, n)$ .*

*Proof.* Following similar steps as in the proof of Lemma 1, supermodularity corresponds to the statement:

$$V_1(e, k, n + 1) - V_0(e, k, n + 1) \geq V_1(e, k, n) - V_0(e, k, n) \quad (18)$$

$\square$

**Corollary 1.** *The threshold function on the available energy to serve a user,  $\eta(n, k)$  defined in Theorem 1 is monotonically non-increasing with slot number  $n$ .*

*Proof.* Let  $n \geq 1$  be the first slot index such that the threshold increases from  $n$  to  $n + 1$ . This means the policy chooses to transmit to a user of some type  $k$  at  $n$  while denying a user of the same type at slot  $n + 1$ , for the same starting energy  $e$ . By (18) this policy can be improved by reversing this decision hence cannot be optimal.  $\square$

## C. Suboptimal Solution: Expected Threshold Policy

DP provides an optimal solution for 0/1 dynamic and stochastic Knapsack problem with growing capacity; however its computational complexity increases exponentially in  $N$ , which is consistent with the NP-hardness of the problem[18]. In this section, a computationally cost- effective suboptimal policy called *Expected Threshold Policy* [19], [12] will be adapted to this problem.

First, we define the following bound on the expectation of energy depletion (RHS of (19)) at slot  $n$  if the available energy is  $e$  and expected harvest amount from slot  $n$  till the end of time horizon  $N$  is denoted as  $\sum_{m=n}^{N-1} \mathbb{E}\{Q_m|Q_1^{n-1}\}$ .

$$e + \sum_{m=n}^{N-1} \mathbb{E}\{Q_m|Q_1^{n-1}\} \geq \sum_{m=n+1}^N \mathbb{E}\{w_m x_m\} \quad (19)$$

After stating a bound on the expected energy consumption from slot  $n$  till the end of time horizon in (19), a computationally cost effective suboptimal policy called "Expected Threshold" is proposed in (22) as follows:

$$x_n(k, e) = \begin{cases} 1 & : e \geq \hat{\eta} \\ 0 & : e < \hat{\eta} \end{cases} \quad (20)$$

where

$$\hat{\eta} = \sum_{m=n+1}^N \mathbb{E}\{w_m x_m\} - \sum_{m=n}^{N-1} \mathbb{E}\{Q_m|Q_1^{n-1}\} \quad (21)$$

As it can be seen from (22) and (23), APOM makes a decision to serve a user of type  $k$  appearing in slot  $n$  if the available energy  $e$  at slot  $n$  is greater or equal to threshold

level  $\eta$ .  $\eta$  is stated as the difference between the expected energy consumption for users with higher value and expected energy replenishment from slot  $n$  till the end of horizon  $N$ .

As an example, if the weights are all equal to one and harvest process is IID then, the expected threshold policy becomes:

$$x_n(k) = \begin{cases} 1 & : e \geq \eta \\ 0 & : e < \eta \end{cases} \quad (22)$$

where

$$\eta = (N - n + 1) \left( \sum_{k'=k+1}^K p_{k'} - q \right) \quad (23)$$

and users of type  $k \in \{1, \dots, K\}$  are arranged such that priority of a user  $(v/w)$  increases with increasing  $k$ .

To examine the performance of the expected threshold policy in Section (V), two more different heuristics are also defined as follows:

**Definition 1.** *Greedy policy is a policy that serves an encountered user whenever there is available energy to serve it.*

**Definition 2.** *Conservative policy is a policy that serves only the best user when there is available energy to serve it.*

#### IV. OPTIMIZING THE SERVICE POLICY OF APOM THROUGH A KNAPSACK PROBLEM FORMULATION OVER DETERMINISTIC MODEL

Responding to instantaneous requests of encountered users, APOM has to adopt an efficient and fast decision making strategy as a new user demand appears. In such problems, if a well defined threshold could be stated, then the threshold based decision mechanism gives a satisfactory result in terms of overall performance and computational complexity. Hence, we shall mainly look for threshold based schemes which provably exhibit experimentally strong performance.

##### A. An Online Policy with Deterministic Threshold Method

This section restricts attention to threshold based decision rules, where the values and weights of the encountered users are compared with a time-varying threshold. In addition to time, the threshold may also be a function of the fraction of remaining capacity in the battery. To consider the deterministic online knapsack problem in a threshold based scheme, upper and lower bounds on the user rate, energy requirement and energy harvesting will be assumed, which are not unrealistic considering practical correspondents to these limitations exist.

The value/weight cost efficiency  $(v/w)$ , will be the critical decision metric for each user. The instantaneous threshold is defined as a monotonic increasing function of  $z_n = \frac{\sum_{m=1}^n x_m w_m}{B}$ , the fraction of the capacity used up by the  $n^{th}$  slot. Following [17], where an optimal threshold scheme was developed for the static capacity 0/1 KP, we restrict attention to the case where the value/weight values are upper and lower bounded by two values  $U, L > 0$ , i.e.  $L \leq \frac{v}{w} \leq U$ , and define

the threshold function as<sup>1</sup>:

$$\Psi(z) = \left( \frac{Ue}{L} \right)^z \frac{L}{e} \quad \text{where } L \leq \frac{v}{w} \leq U \quad (24)$$

where  $e$  denotes the natural logarithm. At each slot  $n$ , the value/weight value of the upcoming user is compared with the threshold  $\Psi(z_n)$ . The threshold based decision rule is the following:

Accept user  $n$  provided it does not violate the current remaining knapsack capacity and  $v_n/w_n \geq \Psi(z_n)$ .

For a static KP,  $z_n$  thus the threshold is monotone nondecreasing, which corresponds to being more willing to include users early on, and being very selective as  $z_n$  increases toward 1. In our problem, the knapsack capacity is not static but gets incremented at arbitrary instants, at arbitrary amounts. Extending the above threshold to the two extreme cases of (1) complete information about the increment amounts, and (2) no information about the increment amounts, the fraction  $z_n$  may be defined in the following different ways.

**Definition 3. Monotone Threshold.** Define  $z_{\text{mon},n} = \frac{\sum_{m=1}^n x_m w_m}{B}$ , where  $B$  is the total amount of energy  $B = B_1 + B_2 + \dots + B_J$  collected from all harvests.

As an alternative way to the monotone threshold approach, we define Jumping Threshold as a piecewise monotone function of the current fraction in each energy harvest interval. It utilizes the the amount of energy harvested up to that time instant as denominator of the fraction.

**Definition 4. Jumping Threshold.** For each  $n$ , let  $J(n)$  be the time of the last harvest before time  $n$ . The fraction of filled capacity at time  $n$  is defined as  $z_{\text{jump},n} = \frac{\sum_{m=1}^n x_m w_m}{B_1 + \dots + B_{J(n)}}$ .

Clearly, this second threshold function is monotone nondecreasing between harvest instants, and jumps down whenever a new harvest occurs. As opposed to the latter threshold function which assumes prior knowledge of all harvest amounts over the problem horizon, this one is an online algorithm by construction.

A common success metric for a deterministic online algorithm is its *competitive ratio*, the worst-case ratio of the algorithm's performance to the optimal offline solution under the same input. An online algorithm  $A$  for a user sequence  $\gamma$  that is  $\alpha$ -competitive satisfies the following:

$$\frac{OPT(\gamma)}{A(\gamma)} \leq \alpha, \quad \text{where } \alpha \geq 1 \quad (25)$$

where  $OPT(\gamma)$  and  $A(\gamma)$  are the values obtained from optimal offline algorithm and the proposed online heuristic  $A$  respectively. Having complete uncertainty in the input, the heuristic proposed should build solutions with a competitive ratio better than the worst-case ratio by  $\alpha$ .

**Remark 1.** Under the condition  $\sum_{m=1}^{N_1} x_m w_m \leq B_1$ , Monotone Threshold guarantees a competitive ratio no more than

<sup>1</sup>The form of this threshold function is found through linear programming and shown to achieve an optimal competitive ratio in [17]

$\ln(U/L) + 1$  assuming two energy harvest intervals, i.e.  $k = 2$ .

Extending the proof in [17] to the dynamic capacity case, the competitive ratio derivation reveals that Monotone Threshold provides the same constant competitive ratio which is optimal in the case of online KP with static and presumably high capacity. Next, we will propose different threshold generation methods using different optimization tools, namely genetic algorithms and fuzzy logic.

### B. Threshold method via Genetic Optimization

Genetic Algorithms is a widely applied technique for optimization and search problems, especially NP-hard ones including KPs. Basically, candidate solutions are stochastically selected, recombined, mutated, either eliminated or retained based on relative fitness; even when the original problem is based upon a deterministic model. We propose the implementation of this stochastic approach to our deterministic problem with the twist that the knapsack capacity may also change as solutions evolve toward better ones in time. Thus, generation adaptation and the capacity incrementation need to be jointly taken into account.

To apply GA on a fraction based scheme, a chromosome is chosen as a vector that defines a threshold for each region of values the fraction may take. For this purpose, the values that remaining fraction of capacity ( $z$ ) can take are quantized in the following manner: The range of fraction ( $[0, 1]$ ) is divided into equal regions as  $[t_1 t_2 \dots t_{1000}]$ , where  $t_i$  corresponds to the threshold for region  $i$ , i.e.  $\psi(z) = t_i$ , and note that  $z \in [\frac{i-1}{1000}, \frac{i}{1000}]$ . A quantization over 1000 intervals is quite sufficient, providing an opportunity to sweep over a wide range. A number of chromosomes are randomly generated at the beginning and their corresponding competitive ratios are found through the fitness function evaluation. The fitness function checks the energy constraint on the available capacity at each step as well. In addition, capacity is updated at each energy replenishment, so is the fraction  $z$ . The observations on the fraction based method on the natural selection of the best users over generations produce a certain competitive ratio in the best and the worst cases for randomly generated parent sequences, provided and discussed in Section V.

### C. Threshold method via Rule Based Optimization

A connected set of well defined rules, consisting of related variables in both the propositions and consequences, can handle uncertain knowledge successfully in decision problems. Although rule based approaches have been implemented in quite a few resource allocation problems in the literature [5], we have come across no previous studies on the threshold determination via this method.

There are two input memberships functions (MF) assigned to define the decision strategy in each possible case for APOM. Both of the input MFs are defined as trapezoidal of 5 degrees. The output MF is assigned as the desired change in the threshold, the ultimate trend of which will be used to determine which users to serve eventually. One of the input membership functions is chosen as the closeness to energy

TABLE I  
MEMBERSHIP RULES OF 5 DEGREES FOR THRESHOLD DETERMINATION  
BELONGING TO THE MEMBERSHIP FUNCTIONS

Energy Harvest Closeness	Capacity Fullness	Threshold
Very-Near	Very-High	Med
Very-Near	High	Low
Very-Near	Med	Low
Very-Near	Low	Very-Low
Very-Near	Very-Low	Very-Low
Near	Very-High	High
Near	High	Med
Near	Med	Low
Near	Low	Very-Low
Near	Very-Low	Very-Low
Med	Very-High	High
Med	High	Med
Med	Med	Med
Med	Low	Low
Med	Very-Low	Very-Low
Far	Very-High	Very-High
Far	High	High
Far	Med	High
Far	Low	Low
Far	Very-Low	Low
Very-Far	Very-High	Very-High
Very-Far	High	Very-High
Very-Far	Med	High
Very-Far	Low	Med
Very-Far	Very-Low	Low

harvest instants in terms of the number of user arrivals. This parameter is prominent in real life scenarios since expecting an energy harvest sooner or at a far instant may completely alter action to be taken at that slot. Once, the harvest instant gets closer and closer, the service provider should adopt a greedy attitude since it would serve as long as its service capacity allows it to do. This metric is chosen to vary between  $[0, 1]$  where the values closer to 1 denotes that an energy arrival is presumed to happen soon, presented as *Very-Near*. Similarly, *Very-Far* stands for the user arrivals at the beginning of an energy harvest interval where the input MF is set to be in the vicinity of 0. In addition to the energy replenishment rate, the fraction of the utilized energy of available capacity is a critical measure as well. Thus, the second MF is assigned as the depletion of available energy of APOM. The values vary between  $[0, 1]$  interval same as the first MF function, ranging from *Very-Low* to *Very-High* in 5 levels.

The behaviour of the threshold function using the input MFs and following the well calibrated rules is shown in Figure 2. It should also be noted that the improved performance of this heuristic is largely related with the enlarged problem dimension. The accuracy of decisions leads to an improved utility maximization performance through proposing a 3D solution to a 2D problem, obviously at an increased complexity.

## V. NUMERICAL AND SIMULATION RESULTS

### A. Stochastic Service Policy Optimization Related Results

It is observed that both the optimal and suboptimal policies behave more conservatively at the beginning, and become more greedy as the end of the problem horizon gets closer. As

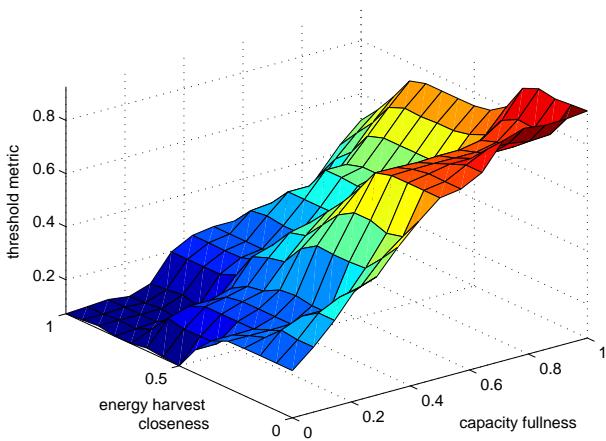


Fig. 2. Surface Graph of Threshold Function Attained via Rule Based Algorithm Described in Section IV-C Indicating the Behaviour of Output Threshold Function with respect to Input Membership Functions

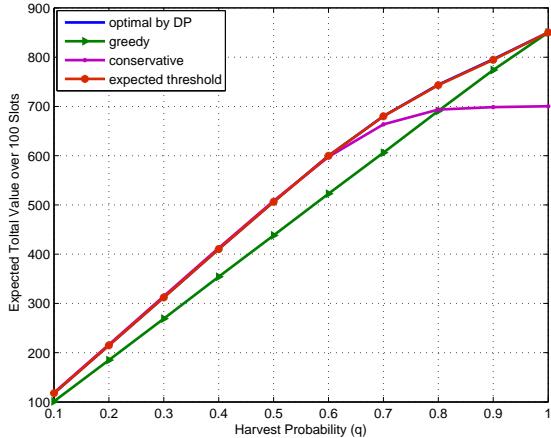


Fig. 3. The Comparison of the Performances for Expected Energy Threshold Policy, Greedy Policy and Conservative Policy wrt. Optimal Policy when available energy=5,  $N = 100$ ,  $K = 2$  for two different user types with efficiency ratios 10 and 5 (best users appear with high probability e.g. 0.7)

illustrated in Figures 3 and 4, the Expected Threshold Policy performs very close to optimal. Drawbacks of purely greedy and conservative policies are also evident on the figures. Figure 3 that when the efficient users appear with high probability, conservative policies outperform greedy ones. On the other hand, when the inefficient users appear with low probability, greedy policies are more advantageous than the conservative approaches. However, Expected Threshold Policy proposed in this paper is robust to variations in user distributions.

#### B. Deterministic Service Policy Optimization Related Results

It will be interesting to study the competitive ratios of the various policies analysed in the previous sections. As a benchmark, the offline optimal policy will be used, hence the values obtained will be overestimating the competitive ratio with respect to the online optimal for each case. In other

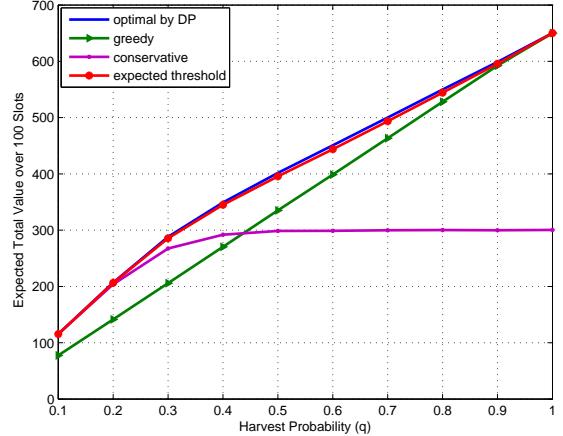


Fig. 4. The Comparison of the Performances for Expected Energy Threshold Policy, Greedy Policy and Conservative Policy wrt. Optimal Policy when available energy=5,  $N = 100$ ,  $K = 2$  for two different user types with efficiency ratios 10 and 5 (worst users appear with high probability e.g. 0.7)

words, the competitive ratios of the algorithms may actually better than the estimates found here.

The simulation results shown in Tables II and III are obtained for the case of predicted energy harvests. User efficiency ( $v/w$ ) ratios are bounded as  $6 = L \leq \frac{v}{w} \leq U = 10$ . User efficiency ratios take uniformly distributed random values within this interval. The knapsack capacity (energy available at the start of the horizon) is 2000. 1000 Monte Carlo trials are conducted, generating 1000 user arrivals on each trial. Results illustrated in Tables II and III show that even the worst-case competitive ratio never exceeds 1.75. Moreover, the results for the monotone threshold function are consistent with the worst possible competitive ratio stated in Section IV-A. Among the tested algorithms, the rule based threshold method has the strongest performance, achieving the lowest worst case competitive ratio.

Next, the energy harvest patterns are considered in a more realistic scenario where the overall resource allocation problem is examined over 10 energy harvests, assumed to occur in a 24-hour cycle. Also, distinct amounts of the harvests are assumed in this case and assigned arbitrarily to model the potential weather condition changes and different locations of the APOM. The competitive ratio analysis for all of the threshold function methods proposed above yield the results shown in the performance graph 5. The worst-case results illustrated in Figure 5 reveal that the Monotone Threshold function and Rule Based Threshold function present closer performance to each other as the user value/weight characteristics are more diverse. However, the rule based threshold provides the best competitive ratio for a less distinct user set as the user diversity ratio ( $U/L$ ) approaches to 0.9 in the worst-case analysis.

## VI. CONCLUSION

We addressed the problem of online user admission for an "access point on the move" scenario. As the energy of the access point gets replenished at arbitrary time instances,

TABLE II  
COMPARISON OF COMPETITIVE RATIOS FOR DIFFERENT THRESHOLD GENERATION METHODS FOR 1000 USERS AND CAPACITY=2000

Threshold method	Average comp. ratio	Worst comp. ratio	Best comp. ratio
Monotone threshold	1.1084	1.3100	1.0640
Jumping threshold	1.3700	1.7200	1.3500
GA based threshold	1.1422	1.5102	1.1087
Rule based threshold	1.0362	1.2066	1.0229

TABLE III  
COMPARISON OF PERFORMANCES FOR DIFFERENT THRESHOLD GENERATION METHODS WITH OPTIMAL OFFLINE SOLUTION FOR 1000 USERS AND CAPACITY=2000

Method	Average tot. value	Worst tot. value	Best tot. value
Offline optimal soln.	17599	17167	18050
Monotone threshold	15880	13374	16647
Jumping threshold	12778	10221	13103
GA based threshold	15416	11581	16042
Rule based threshold	17003	14524	17163

the problem can be modeled as a Knapsack Problem (KP) with dynamic and incremental capacity. We investigated the problem under two different setups where energy and user arrivals are modelled stochastically as well as deterministically. The optimality and structure of a threshold based solution to the stochastic problem was shown, and a computationally friendly "Expected Threshold Policy" was shown to well approximate the optimal DP solution. On the deterministic side, we considered adaptive threshold based policies where a user is admitted if its utility to weight ratio exceeds a certain threshold which may be static or dynamic. A competitive ratio was exhibited for a monotone threshold for the two-harvest case. In addition to extended online threshold functions based on a previous literature, threshold functions using Rule Based approach and a Genetic Algorithm are also developed. Experimental results demonstrate that the proposed decision methods using different threshold functions for the resource allocation problem of the APOM are efficient in achieving close to optimal competitive ratios as well as low computa-

tional complexity. All the proposed scalable solutions can be applicable to other instances of online KP with incremental capacity.

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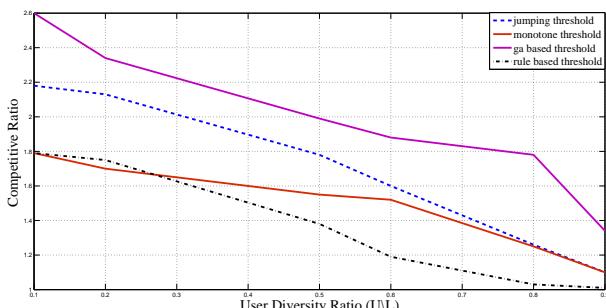


Fig. 5. Performance Evaluation of Different Online Threshold Heuristics vs. Diversity in Users Characteristics: Worst Case Competitive Ratio Analysis, Monte Carlo Simulation of 1000 runs over a Randomly Generated User Sequence of  $N = 1000$  Users under  $J = 10$  Energy Harvests of Different Amounts Modelled over 24-Hour with APOM Capacity Constraint of 2000